The Informal-Formal Interlock: Evidence from the Indian Business Cycle

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Abstract

The informal economy accounts for roughly one-third of GDP and two-thirds of employment in emerging markets. Despite this, only a few studies have investigated the mediating effects of the informal economy on the business cycle. Does the presence of informal labour dampen or amplify the impact of shocks to the business cycle? In this paper, we utilise the Dynamic Stochastic General Equilibrium (DSGE) model developed by Neumeyer and Perri (2005) and extend it to incorporate the informal economy. Then, we test the model using India as a case study. First, we calibrate the model using key parameters of the Indian economy, and then perform an empirical evaluation. This evaluation, which utilises the results of the DSGE model for identification purposes, employs a Mixed Frequency Bayesian Structural Autoregressive (MF-BVAR) approach to assess the role of the informal economy in the Indian business cycle. We find that the existence of the informal economy curbs the impact of most shocks on the business cycle, acting as a safety net. Contrary to the standard view that informality is detrimental to emerging countries, the evidence presented in this study suggests that, at least in the short term, the picture is more complex. On one side, informality can have a positive role by stabilising the domestic economy after international shocks. On the other, a productivity shock to the informal economy represents an additional source of instability that, albeit smaller than that of formal productivity, increases country risk.

Keywords: Informal Economy, Real Business Cycle, Indian economy, Bayesian Var

JEL Classification: C32, C60, E24, E26, E32

1 Introduction

India is one of the world's fastest-growing major economies, with a GDP of 3.1 trillion US dollars in 2021 (S. Gupta 2022). India, similar to many developing countries[1], has a significant informal sector[2] (Murthy 2019). Informal sectors often exist because of a lack of knowledge of the procedures, or people are too poor to bear the costs associated with becoming part of the formal sector[3]. The income of many workers in the informal sector is highly variable, making them vulnerable to economic shocks. Informal firms are often small-scale (Edusah 2013) and lack access to credit and formal financial services, making it difficult to invest in new capital and expand their operations. The most common causes of labour participation in the informal economy are the tax burden, labour market regulation, and the social welfare system (Schneider and Enste 2000, Nguimkeu and Okou 2019). Informality also increases poverty inequality, following Unni and R. Naik (2013). The labourers themselves suffer because of limited access to welfare security, and representation and are extremely vulnerable (Harriss-White 2003). Their lack of representation in mainstream literature exacerbates their marginalisation.

While this sector is often overlooked in discussions of business cycles, it is important to consider the role the informal sector plays in the overall economy as this particular sector has a significant impact on macroeconomic indicators such as GDP and employment. Changes in formal-informal employment can have ripple effects throughout the economy[4] (Altman 2008, Meagher 2013, Valodia

[1]Developing countries has a bigger informal market compared to developed countries (Schneider, Buehn, and Montenegro 2010).

[2]Informal sector includes unregistered businesses (Harriss-White 2003), street vendors (Pilz, Uma, and Venkatram 2015), and home-based workers (Tipple 2005), among others. It is important to note that all markets outside the formal sector are not informal markets (Lippert and Walker 1997). For example, a black market is not the same as an informal economy. In the case of the informal sector, labourers may be involved in a legitimate business but are not registered with the government and thus are exempt from the benefits and rights associated with it.

[3]A. K. Naik (2009) considers there is a significant positive correlation between informal sector employment and the incidence of poverty in Indian states.

[4]Jütting, Parlevliet, and Xenogiani (2008) state that there is a non-negligible number of individuals who move between formal sectors and informal sectors which must be considered in research in the business cycle. During an economic downturn, formal businesses may lay off workers or reduce investment. This can lead to an increase in informal employment as workers turn to self-employment (Picot and Heisz 2000) or informal jobs to make ends meet (Chowdhury, Islam, and Lee 2013). At the same time, informal businesses may be more resilient to economic shocks (Nel, Smart, and Binns 2017), as they are often able to operate with lower costs (Gerxhani 2004) and more flexible labour arrangements (Di Porto, Elia, and Tealdi 2016). This can lead to a shift in economic activity towards the informal sector during a downturn, which can have implications for overall economic growth. Jütting, Parlevliet, and Xenogiani (2008) states it may be that an economic boom encourages labourers to move to the informal sector given tight labour conditions and start their informal businesses. Thus, the relative demand and supply of formal to informal labour is important to consider because it can affect the overall economic performance of a country (Rauch 1991). The ratio of formal to informal labour can have a significant impact on labour market outcomes such as wages, productivity, and labour force participation rates (Carneiro and Henley 2001). K. Gupta and T. Basu (2004) show, through general equilibrium modelling, that under certain conditions, the presence of an informal sector plays a role in the impact of

and Devey 2010, Abid and Salha 2013). As such, it can be useful for policymakers to consider the informal sector when formulating economic policies. These policies can help ensure that the Indian economy remains stable and resilient, even in the face of external shocks and economic downturns. However, given the lack of data and, subsequently, research in this area, it can be difficult for policymakers to track changes affecting informal labourers and implement policies that target them (Alderslade, Talmadge, and Freeman 2006). C. Elgin and Erturk (2019) conduct a comprehensive literature survey on informality and suggests that, even though the literature on informality is growing, future research efforts are very much needed, both on the theoretical and empirical side.

This gap in the literature becomes even more apparent with the emergence of a new form of labour informality in India. Hammer (2019) states that almost half of formal sector employment in India consists of informal workers. Abraham (2017) shows that employing workers in formal enterprises in India without the benefit of employment-related social security has been increasing and is a recent form of informalisation of the labour market. Abraham (2017) uses NSS 68th round data to show that in India, between 1999-2000 and 2011-12, formal labour employment is falling and informal labour employment in formal sectors is increasing. Following Economic Survey of 2021-2022, Government of India, the share of informal workers in the organised sector increased to 54% in 2017-2018. This duality challenges traditional categorisations of formality and informality. This type of informal employment is especially neglected in business cycle studies for India.

A literature review shows formal and informal productivity shocks can have an important impact on the performance of the informal economy[5]. Gupta (2025) shows country risk depends negatively on levels of productivity for India. These findings raise an important question: does country risk[6] respond differently to informal productivity compared to formal productivity shocks?

external factors on the welfare of a developing country.

[5]Informal productivity shocks refer to unexpected changes in the productivity levels of the informal sector. M. Shah and Steinberg (2017) state that crops in India are seasonal and rainfall shocks can be considered important negative productivity shocks. These events can have a huge impact on the productivity of the informal sector, especially for those engaged in agriculture and other rural activities. Recent labour reforms in India, such as the introduction of fixed-term employment contracts, may have a negative impact on the productivity of informal labourers (Jain, Ghosh, and Misra 2023). Technological advancements can also lead to informal productivity shocks. R. Bhattacharya (2019) argues that positive digital productivity shocks can lead to an improvement in the informal economy under certain circumstances. The COVID-19 pandemic, modelled by Fornaro and Wolf (2020) as negative productivity shocks disrupt supply chains (Pujawan and Bah 2022), reduce consumer demand (Bakar and Rosbi 2020, Rahmanov, Mursalov, and Rosokhata 2021, Kanupriya 2021), and lead to job losses (Crayne 2020), which negatively impacts the productivity of informal workers and firms. Guo et al. (2022) analyse the effect of COVID-19 on the informal economy in China and find similar results. Factors such as limited access to credit (Kar and Dutta 2014), lack of formal recognition (Warnecke 2016), and weak infrastructure (Dangra 2016) can have an impact on productivity shocks in the informal sector in India.

[6]Country risk refers to the potential economic, political, and social risks associated with doing business in that particular country. High levels of country risk can make it more difficult for informal businesses to access credit, invest in new equipment or technology, and expand their operations (Kuzilwa 2005). Additionally, political instability

Furthermore, the results of Gupta (2025) demonstrates that real interest rate shocks and country risk shocks play a pivotal role in shaping India's business cycle. Xu, Lv, and Xie (2018), utilising annualised panel data from 131 countries spanning 1999–2007, identify country risk as a robust and significant determinant of the informal economy, motivating to look into country risk and the informal sector in the Indian context. Building on this, we aim to investigate how fundamental shocks from the formal and informal sectors interact with country risk shocks and real interest rate shocks to influence India's informal economy.

In this paper, we examine the impact of country risk, formal and informal productivity and real interest rate shocks on the informal sector of India. We capture the theoretical analysis for India by extending the Dynamic Stochastic General Equilibrium (DSGE) model of Neumeyer and Perri (2005) to include the informal sector in the model. We further conduct an empirical analysis for the informal sector of India for the dataset of 1996Q2-2016Q4 using the Mixed Frequency Bayesian Structural Autoregressive (MF-BVAR) model whose identification is based on the results of the DSGE analysis.

This chapter deviates from existing literature in the following ways: First, the literature on informal sector in the Indian economy is notably sparse. The limited studies available primarily focus on self-employment or agricultural informality, neglecting the "informalisation of the formal sector," a particularly prevalent form of informality in India as highlighted by Hammer (2019) and Abraham (2017). This chapter bridges this gap by extending the DSGE model of Neumeyer and Perri (2005) to incorporate this unique form of informality and provide a nuanced perspective on the formal and informal labour market in India. Secondly, there is a scarcity of research examining the role of the informal sector in India's business cycle dynamics. We bridge this gap by analysing the decomposition of real interest rate shocks and their impact on the Indian informal sector. Country risk and volatility of the real interest rate in India are high at the time of writing, when there have been record corporate bond defaults (Patil and N. Acharya 2021), high internal political instability and social unrest (Alam 2020, Bhatia and Gajjala 2020), worsening international political relationships with Pakistan (Ahmed 2020, Saddiqa and Yousafzai 2019) and China (Miao, D. Huang, and Y. Huang 2023), and two waves of COVID-19 (Dev and R. Sengupta 2020).. The study draws on the findings of Xu et al. (2018), which establish a robust link between country risk and the informal economy globally, to explore this relationship within the specific context of India's business cycle. Thirdly, to the best of our knowledge, this will be the first study that uses

or social unrest, which is an issue in India (Alam 2020, Bhatia and Gajjala 2020, Ahmed 2020, Saddiqa and Yousafzai 2019, Miao, D. Huang, and Y. Huang 2023), can lead to disruptions in supply chains or transport (Wagner and Bode 2008), making it more difficult for informal businesses to operate. Country risk factors, such as corruption, which is high in India, following "Corruption Perceptions Index: India" (2022), or climate risk, following Mandal and A. C. Gupta (2017) can exacerbate the challenges faced by informal businesses.

the Mixed Frequency Bayesian VAR model to study the informal sector in India. This innovative methodological approach enables the integration of mixed-frequency data into a macroeconomic framework, offering a more precise understanding of the impact of shocks on informal sectors in India.

Let us recall the benchmark DSGE model of Neumeyer and Perri (2005). Firms use a working capital loan to pay labour a part of their wages before production. Firms pay back the loan with interest at the end of production. The real interest rate involved in paying back the loan is modelled as a product of country risk and foreign real interest rate. Country risk is modelled in two ways: an independent country risk model where the country risk process follows a stochastic process and an induced country risk approach where country risk depends on productivity levels.

We extend the model of Neumeyer and Perri (2005) to incorporate the informalisation of the formal sector in India. We assume the formal firm hires both formal and informal labourers. Firms have a contract with the formal labour that firms need to pay formal labour a portion of their wages before production. There is no such contract with informal labourers. Firms take a working capital loan to pay formal labour a portion of their wages at the beginning of production. Firms pay back the working capital loan with interest. The real interest rate associated with the working capital loan is modelled further as a product of country risk and the real interest rate. We model country risk in three ways: (1) independent country risk model where country risk process follows an AR(1) model; (2) formal productivity induced approach where country risk depends on formal productivity; (3) informal productivity induced approach where country risk depends on informal productivity. Furthermore, formal labourers pay tax, while informal labourers do not. We add a government sector, where the government earn money through income taxes.

In this chapter, we add to the original model of Neumeyer and Perri (2005) in the following ways: (1) inclusion of formal and informal labour with different wage rates; (2) inclusion of the contract with formal labour which states firms need to pay formal labour a portion of wages before production, unlike informal labour; (3) modelling of country risk depending on formal and informal productivity separately; (4) inclusion of taxation and the government sector.

The consideration of working capital in the model is an influential component in checking the informality of India[7]. In the model, we have also considered the role taxation and wage inequality plays on how real interest rate shocks impact the formal and informal market[8].

We calibrate the model for India between 1996Q2-2016Q4 and obtain the simulated impulse responses. Table 1 describes the different shocks considered and the expected results.

[7]Mitra (2013) shows informality weakens the working capital channel of financial development.

[8]Schneider and Enste (2013) and Williams and Lansky (2013) state that heavy intervention by the state in form of taxation can lead to more individuals in the informal economy.

Shocks Name	Description	Expected Impact
Country Risk Shocks (Independent Country Risk Approach)	Country Risk Process follow an AR(1) model and do not interact with any other shocks. We look into the impact of country risk shocks on the Indian informal sector.	Country risk shocks have a positive impact on the informal sector, following Xu, Lv, and Xie (2018).
Country Risk Shocks (Formal Productivity Induced Approach)	Changes in country risk depend on the formal productivity level of the country. We look into the impact of country risk shocks on the Indian informal sector when the country risk shocks have been caused by a change in the level of formal productivity.	We expect formal productivity shocks to negatively impact country risk. We expect country risk shocks to have a positive impact on the informal sector, following Xu, Lv, and Xie (2018).
Country Risk Shocks (Informal Productivity Induced Approach)	Changes in country risk depend on the informal productivity level of the country. We look into the impact of country risk shocks on the Indian informal sector when the country risk shocks have been caused by a change in the level of informal productivity.	We expect informal productivity shocks to positively impact country risk (intuition from Schneider and Enste 2000). We expect country risk shocks to have a positive impact on the informal sector, following Xu, Lv, and Xie (2018).
Formal Productivity Shocks (Independent Country Risk Approach)	Formal productivity shocks have no spillover impact. When formal productivity rises, country risk is not impacted. We look into the impact of formal productivity shocks on the Indian informal sector when no other shocks interact with formal productivity.	Formal productivity shocks have a negative impact on the informal sector (Neumeyer and Perri 2005).
Formal Productivity Shocks (Formal Productivity Induced Approach)	We look into the impact of productivity shocks on the Indian informal sector when the rise in productivity causes a change in country risk level.	Formal productivity shocks have a negative impact on the informal sector of India, but the impact should be magnified due to spillover effects to country risk and subsequently, real interest rate (Neumeyer and Perri 2005).
Informal Productivity Shocks (Independent Country Risk Approach)	Informal productivity shocks have no spillover impact. When informal productivity rises, country risk is not impacted. We look into the impact of informal productivity shocks on the Indian informal sector when no other shocks interact with informal productivity.	Informal productivity shocks have a positive impact on the informal sector (Fiess, Fugazza, and Maloney 2010).
Informal Productivity Shocks (Informal Productivity Induced Approach)	We look into the impact of informal productivity shocks on the Indian informal sector when the rise in productivity causes a change in the country-risk level.	Informal productivity shocks have a positive impact on the informal sector of India, but the impact should be magnified due to spillover effects to country risk and subsequently, real interest rate (intuition from Schneider and Enste 2000, Neumeyer and Perri 2005, Fiess, Fugazza, and Maloney 2010).
Foreign Real Interest Rate Shocks	We check the impact of foreign interest rate shocks on the informal sector of India.	Foreign real interest rate shocks have a positive impact on the informal sector (Horvath 2018).

We set up the Mixed Frequency Bayesian VAR model (MF-BVAR) to empirically study the impact of the shocks on India's informal sector. The identification of the MF-BVAR models is based on sign restrictions obtained from a DSGE model.

Thus, a major contribution of this chapter is extending the DSGE model of Neumeyer and Perri (2005) to incorporate characteristics of the informal sector relevant to India. A secondary contribution of the chapter is the analysis of the impact of shocks on the informal sector in India using Mixed-Frequency Bayesian VAR with sign restrictions.

The rest of the chapter is organised as follows. Section 2 delves into the literature survey for this chapter. Section 3 explains the DSGE Model. Section 4 analyses the model solutions. We calibrate the values of the model in Section 5 for India. Section 6 shows the simulated impulse responses for India. Section 7 deals with sensitivity analysis. Section 8 explains the empirical methodology of the Mixed Frequency Bayesian VAR model. Section 9 explains the impulse responses obtained from the Mixed Frequency Bayesian VAR analysis. Section 10 concludes the chapter.

2 Literature Review

In this section, we discuss past papers closely related to our topic. First, we investigate relevant theoretical contributions to the body of literature for the business cycle of emerging countries. Alege (2012) develops a Dynamic Stochastic General Equilibrium (DSGE) model to investigate the causes of fluctuations in business cycles of Nigeria and concludes that both real and nominal shocks have a significant impact on the business cycle. Ghazouani, Drissi, and Boukhatem (2019) highlight the importance of financial integration in affecting the volatility of macroeconomic fundamentals using a DSGE model. Aguiar and Gopinath (2007) use a standard small open economy model for both emerging and developed markets, augmented to include transitory and trend shocks. Aguiar and Gopinath (2007) consider Mexico and Canada as case studies and use GMM estimation as a methodology to see how business cycle models differ for developed and developing countries. They find that the volatility of the interest rate in developing countries compared to developed countries is high. They show that emerging market business cycles exhibit strongly countercyclical current accounts, unlike developed countries. Consumption volatility exceeds income volatility in developing countries. They further conclude that shocks to trend growth instead of transitory fluctuations around a stable trend contribute to fluctuations in emerging markets. Their results help bring forward the differences between developed and emerging countries and the importance of researching emerging countries. Neumeyer and Perri (2005) consider a standard one-good neoclassical small open economy to enquire about the impact of various shocks on the business cycle of Argentina. Tiryaki (2012) uses the model of Neumeyer and Perri (2005) to find the

effect of shocks on the real economy of Turkey. We use this model of Neumeyer and Perri (2005) for our analysis. Some other recent contributions in the field of business cycle fluctuations in emerging countries include Challita (2023), Arabi (2020), Lin (2021), Guerra-Salas, Kirchner, and Tranamil-Vidal (2021) and Sobieraj and Metelski (2021).

Given the focal country of our discussion, we look into the theoretical contributions in the field of business cycles for India. Anshul Kumar (2023) develops a closed DSGE model to check the the effectiveness of monetary policy on business cycles in India and finds that monetary policy affects the Indian GDP only in the short run. Goyal and Abhishek Kumar (2018) consider a New Keynesian model to take into consideration the effect of changes in money balances on output and inflation. They find that money balances affect output and inflation significantly, even after correcting for money demand in India, unlike developed countries. This paper points out the important role nominal interest rates play in the Indian economy. Banerjee, P. Basu, and Ghate (2020) construct a New Keynesian monetary business cycle model to study the monetary transmission of India and finds money shocks have a stronger impact on the GDP of India compared to nominal interest rate shocks. Bhat, Kamaiah, and D. Acharya (2020) develop a structural model to estimate the impact of different monetary policies on inflation in India and find that money shocks and nominal interest rate shocks negatively impact the price level in India. Banerjee and P. Basu (2019) utilise a Real Business Cycle model of a small open economy hit by investment-specific technology shocks to look at the business cycle of India. This paper finds that TFP shocks are the foremost contributor to aggregate fluctuations in India. This result motivates us to consider a model that looks into how productivity shocks interact with real interest rate shocks to affect India's business cycle.

From an empirical standpoint, with regards to India's business cycle evaluation, R. Pandey, Patnaik, and A. Shah (2019) analyse the pre-1991 period for India and find that the business cycle is mainly driven by monsoon shocks. They also analyse the post-1991 phase, where they find that India's business cycle is driven by investment-inventory fluctuations. Paramanik, Bhandari, and Kamaiah (2022) attempt to assess the relations between the business cycle, financial cycle, and economic uncertainty for India during the period January 2003 to January 2020 and conclude that there is a bidirectional causal relationship between the real and financial markets. Arora (2018) attempts to find the impact of monetary and fiscal policies on select macroeconomic indicators from 1990Q1 to 2011Q4 using Structural Variance Analysis. However, the macroeconomic indicators they considered are limited to GDP, inflation, short-term interest rate, government expenditure and real exchange rate. These variables are available quarterly in India and have been researched on. Most research in India that conducts analysis with quarterly data focuses on these variables and ignores labour due to lack of data. This paper has attempted to bridge the research gap. Labour is an important aspect of the business cycle of India given the labour-driven nature of the

economy. Rand and Tarp (2002) consider India, Korea, Morocco, Pakistan, Malaysia, Uruguay, Colombia, Peru, Chile, Mexico, South Africa, Malawi, Nigeria, Cote d'Ivoire and Zimbabwe and demonstrate that developing countries differ considerably from their developed counterparts in terms of characteristics of short-run macroeconomic fluctuations. They find that the stylized facts of the business cycle in emerging countries differ from those of developed countries, which further motivates the research of this thesis.

The literature review conducted so far provides us with some insights. Firstly, almost all DSGE models of most emerging countries, especially India focus on monetary and fiscal policies. Secondly, the analysis of the business cycle differs for developing countries compared to developed countries.

We look at the strand of literature that has focused on the role of informal sector in business cycle analysis from a theoretical as well as empirical perspective. Marjit, Bhattacharyya, and L. Yang (2023) use the general equilibrium model to show that during severe demand and supply shocks, the informal sector in emerging countries reacts the opposite to the formal sector, mitigating the overall effect of the shocks. Fernández and Meza (2015) build a DSGE model of a small open economy and calibrates it for Mexico to show imperfect measurement of the informal sector[9] can lead to stronger variability in the business cycle. Restrepo-Echavarria (2014) introduces an informal sector into a small open economy DSGE model and shows that the informal sector, poorly measured in national accounts, generates volatility of measured consumption that is higher than that of output. Lahcen (2014) consider a DSGE model for Morocco that includes trade and informality and utilises Bayesian estimation to show that the presence of the informal sector absorbs some of the effects of productivity shocks on the whole economy while there is no such evidence for foreign demand shocks. Horvath (2018) uses DSGE model analysis to show real interest rate shocks lead to contraction of formal sector and expansion of informal sector in Argentina and Mexico. However, their intuition of the informal sector in their DSGE model is different from our model. While we tailor our model to reflect the informalisation of formal sector, Horvath (2018) models two distinct formal and informal sectors. C. Elgin et al. (2012) considers a panel of 152 countries from 1950 to 2000 and uses dynamic general equilibrium model to show that the presence of informal sector increases the amplitude of business cycles. Ferreira-Tiryaki (2008) uses the general method of moments methodology of data spanning 1985 to 2002 for 24 countries to find that countries with higher presence of informal sector have more volatile business cycles. Most literature on informal sectors in business cycle analysis has been conducted theoretically using DSGE or general equilibrium models. Fewer studies have been conducted using empirical methodologies, which

[9]They model the 'self-employment' component of the informal sector.

can be attributed to the lack of data. We solve this problem by using the recent database obtained by Elgin et al. (2021) using Dynamic General Equilibrium estimates (details in Section 8.1).

For the next line of literature, we look at papers that have the informal sector in India in business cycle analysis. Gabriel et al. (2011) use a New Keynesian dynamic stochastic general equilibrium (DSGE) model for India and estimate it using Bayesian Maximum likelihood methods. The paper finds the distinction between formal and informal markets and the inclusion of financial frictions leads to a better empirical performance of business cycle models. Chakrabarti and Kundu (2009) construct a multi-sector macroeconomic model to examine changes in agriculture policy and fiscal policy changes on the formal and informal economy in India. They find fiscal shocks lead to the expansion of the informal sector in India. S. Bhattacharya and Kesar (2018) use statistical inference to find that there exists a dualism between capitalist and non-capitalist spaces within the informal sector in India. There has not been much work on the informality of India in macroeconomics literature, especially when it comes to informal-formal employment, the type of informality mentioned in Hammer (2019) and Abraham (2017). This has created a gap in literature, which this paper aims to fill.

We focus on papers that have worked on Mixed-Frequency Bayesian Vector Auto Regression (MF-BVAR) Models. Schorfheide and Song (2021) use the Mixed Frequency Bayesian VAR model to generate macroeconomic forecasts for the USA during COVID-19. Huber et al. (2023) utilise Mixed Frequency Bayesian VAR and proves that this methodology is better suited for nowcasting models faced with extreme observations compared to Mixed Frequency Linear VAR models. Foroni, M. G. Marcellino, and Stevanovi (2018) and Gefang, Koop, and Poon (2020) discuss different mixed frequency models one can use for estimation. Bai, Ghysels, and Wright (2013) check the relationship between MIDAS regressions and Kalman filter state space models. Koop, Korobilis, et al. (2010) postulates the basic foundations of Bayesian VAR. The papers mentioned help identify Bayesian methodology to be the best fit for our mixed frequency model[10]. Brave, Butters, and Justiniano (2019) conduct a mixed frequency Bayesian VAR forecast for the US and demonstrate the importance of choosing specifications for models. Ankargren and Jonéus (2019) address uses mixed-frequency Bayesian VAR with US data where GDP growth is the sole quarterly variable, with the remaining being monthly variables. Schorfheide, Song, and Yaron (2018) use an application of mixed frequency Bayesian VAR model for their research which requires high-frequency monthly data as the available monthly data is contaminated by measurement error.

We find that research conducted on informality in India is not in the business cycle context (Mehrotra and Giri 2023, Bahl and Sharma 2024, Kesar 2023, Kesar et al. 2021). The few papers

[10]Ferroni and Canova (2021) explain MF-BVAR programming in MATLAB, while Ankargren and Y. Yang (2019) does the same for R.

that focus on macroeconomic issues in India do not include the impact of interaction among various fundamental shocks on the informal sector of India. We cannot find a comprehensive DSGE model investigating the impact of the aforementioned shocks on the Indian informal economy, keeping in mind taxation, wage inequality and working capital for India. Secondly, no paper has focused on the impact of macroeconomic shocks in the scenario of formal sectors hiring informal workers in India. Thirdly, there is no work using mixed-frequency Bayesian VAR with sign identification in the informal sector of India. These gaps in literature lead us to enquire regarding the impact the interaction of country risk shocks, foreign real interest rate shocks, formal productivity shocks, and informal productivity shocks have on informal employment within the formal economy and its informal counterparts in India.

3 Model Description

In this section, we discuss the DSGE model. We construct a standard one-good neoclassical small open economy model inspired by the original model of Neumeyer and Perri (2005).

Time is discreet. t^- refers to the beginning of the period before production begins while t^+ refers to when production ends. The firms employ two types of labour: formal labour and informal labour. We define formal labourers as the labourers who have established a formal contract with their firms that states the firm has to pay a portion of their wages in time t^-.

In time t^-, firms hire formal labour l_{f_t}, informal labour l_{i_t} and capital k_{t-1}. In time t^+ formal output y_{f_t} is produced and informal output y_{i_t}. Firms are required to pay informal labour a total amount of $l_{i_t} w_{i_t}$ and formal labour a total amount of $l_{f_t} w_{f_t}$ by t^+.

A non-contingent real bond is traded in international financial markets. The firms need this working capital to pay their formal labourers a fraction of their wages before the production ends, based on their contract. θ is their working capital parameter. Since the contract does not apply to informal labourers, we assume the firms are not obligated to pay informal labour in advance. Firms pay the formal labourers $\theta l_{f_t} w_{f_t}$ in time t^-, which they borrow from the financial market. Firms have to pay back the working capital loan plus the interest $R_{t-1}\theta l_{f_t} w_{f_t}$ in time t^+ and pay the remaining wage to the formal labourers $(1-\theta) l_{f_t} w_{f_t}$. R_t is the payment of loan as well as interest, the net interest is given by $R_t - 1$.

Households set up preferences with formal consumption, informal consumption, formal labour and informal labour and maximise utility subject to budget constraints. The government earns revenue by imposing a time-invariant tax on income. Only formal labourers pay tax.

We add to the original model of Neumeyer and Perri (2005) in the following ways: (1) inclusion of formal and informal labour with different wage rates; (2) inclusion of a contract with formal

labour which states firms need to pay formal labour a portion of wages before production, unlike informal labour; (3) modelling of country risk depending on formal and informal productivity separately; (4) inclusion of taxation and the government sector.

We set up the model for households, firms, and government, followed by the decomposition of the real interest rate.

3.1 Households

In this subsection, we describe the household side of the model.

We assume the consumer has a preference over sequences of formal consumption, informal consumption, formal labour and informal labour and maximises expected utility. To represent consumer behaviour, we consider a modified version of the Greenwood–Hercowitz–Huffman (GHH) Utility given by Equation 1.

$$u(c_{f_t}, c_{i_t}, l_{i_t}, l_{f_t}) = \frac{1}{1-\sigma}[c_{f_t} + c_{i_t} - \psi_{\ 1} l_{i_t}^{\nu_1} - \psi_2 l_{f_t}^{\nu_2}]^{1-\sigma} \tag{1}$$

where σ is the coefficient of risk aversion, ν_1 is the inter-temporal elasticity of informal labour supply, ν_2 is the inter-temporal elasticity of formal labour supply, ψ_1 is the informal labour weight, ψ_2 is the formal labour weight, c_{f_t} is formal consumption and c_{i_t} is informal consumption.

This utility function is constructed using GHH preferences by Greenwood, Hercowitz, and Huffman (1988) and Restrepo-Echavarria (2014). The GHH utility function, as highlighted by Neumeyer and Perri (2005), is best for capturing the essence of the business cycle models because labour supply is to be independent of consumption.

The budget constraint is given by:

$$c_{f_t} + c_{i_t} + x_t + b_t + \kappa(b_t) \leq l_{i_t} w_{i_t} + (1-\tau) l_{f_t} w_{f_t} + R_{t-1} b_{t-1} - r_t k_{t-1} \tag{2}$$

where x_t is investment, b_t is bonds, and $\kappa(b_t)$ is a convex function and denotes the cost of holding bonds.

The budget constraint states the total spending side, consisting of consumption, investment, bonds holding at time t and the cost of holding bonds should be less than or equal to the total income from capital, formal labour, informal labour and bonds holding at time t-1.

The capital accumulation equation is given by:

$$x_t = k_t - (1-\delta)k_{t-1} + \Phi(k_{t-1}, k_t) \tag{3}$$

δ represents the depreciation rate.

$\Phi(k_{t-1}, k_t)$ is the adjustment cost in order to avoid excessive volatility of investment and is given by:

$$\Phi(k_{t-1}, k_t) = \frac{\phi}{2} k_{t-1} \left[\frac{(k_t - k_{t-1})(1+\gamma)}{k_{t-1}} \right]^2 \tag{4}$$

where ϕ represents the capital adjustment cost parameter.

Bond holding cost is given by:

$$\kappa(b_t) = \frac{\kappa}{2} y_t \left[\frac{b_t}{y_t} - \bar{b} \right]^2 \tag{5}$$

κ denotes the size of the bond holding costs and $\bar{b}$ is the the ratio of bonds to GDP at the steady state.

The Lagrangian is given by:

$$L = \frac{1}{1-\sigma} [c_{f_t} + c_{i_t} - \psi_1 l_{i_t}^{v_1} - \psi_2 l_{f_t}^{v_2}]^{1-\sigma} +$$
$$\lambda_{f_t} [l_{i_t} w_{i_t} + (1-\tau) l_{f_t} w_{f_t} + r_t k_{t-1} + R_{t-1} b_{t-1} - c_{f_t} - c_{i_t} - x_t - b_t - \kappa(b_t)] \tag{6}$$

3.2 Firms

In this subsection, we describe the firm side of the model.

Each firm produces its final output by using formal and informal sectors as modelled by Busato and Chiarini (2004). Following Pappa, Sajedi, and Vella (2015), one singular firm employs formal workers as well as informal workers to produce output[11], which reflects the rapid informalisation of the formal sector in India. Firms employ informal labour to avoid paying workers wages before production starts as part of the contract. Labour moves freely in this scenario[12].

[11] One might question why a single firm would utilise distinct production functions for formal and informal outputs. Following the framework established by Pappa, Sajedi, and Vella (2015), we model the production of formal and informal outputs separately. Although both outputs are generated by the same firm, the use of separate production functions reflects the differing levels of skills, productivity, and capital requirements typically associated with each type of output. Moreover, this distinction provides a clearer representation of the quantities of formal and informal output produced, offering valuable insights into the firm's production structure. The firms maximise one singular profit equation taking into account both the production functions.

[12] The free mobility assumption is consistent with evidence from the literature on labour market segmentation between the formal and the informal sectors. Magnac (1991) uses an integrated micro-economic model and data on women's labour force participation in Colombia in 1980 to conclude there is free mobility within the labour market. Maloney (1999) studies patterns of labour mobility between sectors in Mexico and finds that there is no significant segmentation between the formal and the informal labour markets. Pratap and Quintin (2006) use data from Argentina's household survey to find that there is no evidence to support "the mainstream view that labour markets are segmented along formal/informal lines in developing nations such as Argentina." Restrepo-Echavarria (2014) constructs a two-sector small open economy business cycle model and assumes labour is free to flow between sectors.

The production functions for formal and informal output are given by:

$$y_{f_t} = A_{f_t} l_{f_t}^{\alpha} k_{t-1}^{1-\alpha} \tag{7}$$

$$y_{i_t} = A_{i_t} l_{i_t}^{\beta} \tag{8}$$

A_{f_t} is the average formal productivity. A_{i_t} is the average formal productivity. y_{f_t} is formal output. y_{i_t} is informal output, k_t is capital, l_{f_t} is formal labour and l_{i_t} is informal labour.

Informal labour is decreasing returns to scale[13], following Busato and Chiarini (2004), Horvath (2018) and Restrepo-Echavarria (2014).

The rental rate of capital is r_t. w_{f_t} is the formal wage rate while the informal wage rate is w_{i_t}.

The profit maximising condition of firms is given by:

$$\Pi_t = y_{i_t} + y_{f_t} - (1-\theta) l_{f_t} w_{f_t} - R_{t-1} \theta l_{f_t} w_{f_t} - l_{i_t} w_{i_t} - r_t k_t \tag{9}$$

3.3 Government

In this subsection, we describe the government sector.

The government imposes a time-invariant income tax of τ on formal wages w_{f_t}. Only formal labourers l_{f_t} pay taxes. Following Horvath (2018), we assume, without loss of generality that government revenue is equal to government spending given a balanced budget condition.

$$g_t = \tau w_{ft} l_{ft} \tag{10}$$

3.4 Real Interest Rate Decomposition

In this subsection, we outline the decomposition of the interest rates with respect to formal productivity, informal productivity, foreign interest rates and country risk.

We assume deviations from trend of formal productivity $\hat{A}_{f_t}$ and informal productivity $\hat{A}_{i_t}$ follow AR(1) processes, given by:

$$\hat{A}_{f_t} = \rho_a \hat{A}_{f_{t-1}} + \epsilon_{A_{ft}} \tag{11}$$

$$\hat{A}_{i_t} = \rho_a \hat{A}_{i_{t-1}} + \epsilon_{A_{i_t}} \tag{12}$$

where the error terms $\epsilon_{A_{f_t}}$ and $\epsilon_{A_{i_t}}$ are normally distributed and and not serially auto correlated.

[13]Lucas Jr (1988) state that if informal production is less capital intensive than formal production, we can set the exponent of capital for informal production to zero without loss of any generality.

We assume that foreign lenders give a loan at a rate of R_t to India. However because it is possible for payments to foreigners to default, loans are considered risky investments. Real interest rate volatility has two main causes. First, the real interest rate fluctuates with risk. Second, while the risk of default remains unchanged, the real interest rate fluctuates due to shifts in foreign investors' inclinations toward riskier assets.

Thus, the real interest rate can be modelled as:

$$R_t = R_t^* D_t \tag{13}$$

R^* is the international rate for risky assets, which we will consider the USA rate of interest and D^* refers to country spread risk.

We can formulate the deviation from the trend of the international rate of interest as an AR(1) process, given by:

$$\hat{R}_t^* = \rho_{R^*} \hat{R}_{t-1}^* + \epsilon_{R^*_t} \tag{14}$$

The error term is normally distributed with no serial auto-correlation.

There are three ways to model the country risk spread D. Firstly, independent factors which are not dependent on either formal or informal productivity shocks can impact country risk levels. We will refer to this scenario as the independent country risk approach. Country risk is given by an AR(1) process in the case of the independent country risk approach, as shown in Equation 15.

$$\hat{D}_t = \rho_D \hat{D}_{t-1} + \epsilon_{D_t} \tag{15}$$

The error term is normally distributed with no serial auto-correlation.

Secondly, fundamental shocks to the economy can affect the business cycle directly and indirectly via country risk at the same time.

The country risk component D_t can be considered to be a diminishing function of formal productivity shocks. We will refer to this approach as the formal induced country risk approach. The representative equation is given by:

$$\hat{D}_t = \bar{\eta}_f (E_t \hat{A}_{f_{t+1}}) + \epsilon_{\eta_{f_t}} \tag{16}$$

$\bar{\eta_f}$ is expected to be negative and it captures the sensitivity of the country risk to formal productivity shock.

The country risk component D_t can be considered to be a function of informal productivity

shocks[14]. We will refer to this approach as the informal induced country risk approach. The representative equation is given by:

$$\hat{D}_t = \bar{\eta}_i(E_t\hat{A}_{i_{t+1}}) + \epsilon_{\eta_{i_t}} \tag{17}$$

$\bar{\eta}_i$ captures the sensitivity of the country risk to informal productivity shock.

4 Model Solution

In this section, we first present the equilibrium equations based on the first-order conditions obtained from the analysis. Secondly, we present the log linearised equations and attempt to explain them. Step-by-step calculations are provided in Appendix A.2.

4.1 Equilibrium Conditions

In this subsection, we discuss the equilibrium conditions of the model. We obtain the first-order conditions of formal and informal labour, formal consumption, and capital from the derivations of the DSGE model and calculate the equilibrium conditions. Step-by-step calculations are provided in Appendix A.2. The equilibrium conditions for investment, government expenditure, formal output, and informal output are taken from Equation 3, Equation 10, Equation 7 and Equation 8 respectively. We set up the equilibrium conditions of net exports and informal consumption, following Horvath (2018).

Following Horvath (2018), formal output produced is in equilibrium with formal consumption, formal investment, government expenditures, and trade balance. On the other hand, informal output is used for informal consumption. This is in line with the empirical evidence of Gasparini and Tornarolli (2009). The two equations are given by:

$$y_{f_t} = c_{f_t} + x_t + g_t + nx_t \tag{18}$$

where nx_t refers to net exports, g_t refers to government expenditure, c_{f_t} refers to formal consumption and y_{f_t} refers to formal output.

$$y_{i_t} = c_{i_t} \tag{19}$$

where c_{i_t} and y_{i_t} are informal consumption and output respectively.

First, we obtain the labour demand curve from the differentiation of the profit equation (Equation

[14]High informal productivity signals growth in economy occurring outside regulatory frameworks (Schneider and Enste 2000) and raises investor concerns, impacting country risk.

9) with respect to formal labour and equalising it to zero (details in Appendix A.2). We assume the sum of the proportion of formal and informal labour equals to a constant[15]. We call the equation representing labour demand as the relative demand equation as it shows the relative demand of formal labour compared to informal labour.

The relative labour demand equation is given by:

$$A_{f_t}\alpha {l_{f_t}}^{(\alpha-1)} k_{t-1}^{1-\alpha} - A_{i_t}\beta {l_{i_t}}^{\beta-1} = [1+\theta(R_{t-1}-1)]w_{f_t} - w_{i_t} \tag{20}$$

where α refers to formal labour share, β refers to informal labour share, θ refers to working capital parameter.

In Equation 20 the term $[1+\theta(R_{t-1}-1)]w_{f_t}$ is the amount the firm spends to pay formal labour, including their wages and the working capital loan needed to afford formal workers[16]. $\theta(R_{t-1}-1)w_{f_t}$ is the net payment the firm pays in the form of a working capital loan.

If $A_{f_t}\alpha {l_{f_t}}^{(\alpha-1)} k_{t-1}^{1-\alpha} - A_{i_t}\beta {l_{i_t}}^{\beta-1}$ is greater than zero, the demand of formal labour is higher than demand of informal labour. Thus, the net amount a firm is willing to pay for formal labour is higher than the net amount a firm is willing to pay for informal labour i.e. $[1+\theta(R_{t-1}-1)]w_{f_t} > w_{i_t}$.

Let us simplify the model and consider the wage differential as constant[17].

$$\frac{w_{f_t}}{w_{i_t}} = \mu \tag{21}$$

where μ is a simplistic way of showing the wage differential between the formal and informal wage.

The redefined relative labour demand curve is given by:

$$A_{f_t}\alpha {l_{f_t}}^{(\alpha-1)} k_{t-1}^{1-\alpha} - A_{i_t}\beta (l_{i_t})^{\beta-1} = [1-\theta(1-R_{t-1}) - \frac{1}{\mu}]w_{f_t} \tag{22}$$

We attempt to interpret Equation 22 in this paragraph. w_{f_t} is always positive. When $[1-\theta(1-R_{t-1}) - \frac{1}{\mu}] > 0$, then $A_{f_t}\alpha {l_{f_t}}^{(\alpha-1)} k_{t-1}^{1-\alpha} - A_{i_t}\beta (l_{i_t})^{\beta-1} > 0$. This implies that when the demand for formal labour is higher than that of informal labour then the firm is willing to pay formal labour a higher amount

[15]Marjit, Bhattacharyya, and L. Yang (2023) states "By its very essence the informal sector is a place where all those who are forsaken by the formal sector find refuge." Therefore following Marjit, Bhattacharyya, and L. Yang (2023) and a simplified version of Pappa, Sajedi, and Vella (2015), when the proportion of formal labour increases, that of informal labour decreases and vice versa.

[16]Recall, firms need to pay formal labour a portion of their wages at the beginning of production. Firms take a working capital loan to afford this payment. Firms need to pay back the loan with interest.

[17]Literature shows that the ratio of formal-informal wage in India has remained relatively constant over the years, as observed from M. Kumar and S. Pandey (2021), Singhari and Madheswaran (2017), Narayanan 2015). Using a constant wage differential makes it more convenient to capture changes in formal-informal labour due to changes in wage differential.

compared to informal labour. The vice versa holds when relative demand for informal labour is positive[18].

We obtain the labour supply curve from the differentiation of the Lagrangian (Equation 96) with respect to formal labour[19] and equalising it to zero (details in Appendix A.2). We refer to the equation depicting labour supply as the relative supply equation as it shows the relative supply of formal labour compared to informal labour.

The relative labour supply is given by:

$$-\psi_1 v_1 l_{i_t}^{v_1-1} + \psi_2 v_2 l_{f_t}^{v_2-1} = (1-\tau) w_{f_t} - w_{i_t} \tag{23}$$

where v_1 is the inter-temporal elasticity of informal labour supply, v_2 is the inter-temporal elasticity of formal labour supply, ψ_1 is the informal labour weight, ψ_2 is the formal labour weight, and τ is the income tax rate.

We input Equation 21 in Equation 23. The simplified relative labour supply equation is given by:

$$-\psi_1 v_1 l_{i_t}^{v_1-1} + \psi_2 v_2 l_{f_t}^{v_2-1} = (1-\tau-\frac{1}{\mu}) w_{f_t} \tag{24}$$

Equation 24 shows if the total compensation to formal labourers is higher than that to informal labourers, the supply of formal labourers is higher than informal labourers and vice versa. We equate to Equation 24 with Equation 25 to get the labour market equilibrium. Since w_{f_t} has to be positive, when $(1-\tau-\frac{1}{\mu}) w_{f_t} > 0$, then $-\psi_1 v_1 l_{i_t}^{v_1-1} + \psi_2 v_2 l_{f_t}^{v_2-1} > 0$.

The equilibrium is given by:

$$\frac{A_{f_t} \alpha {l_{f_t}}^{(\alpha-1)} k_{t-1}^{1-\alpha} - A_{i_t} \beta (l_{i_t})^{\beta-1} k_{t-1}^{1-\beta}}{[1-\theta(1-R_{t-1})-\frac{1}{\mu}]} = \frac{-\psi_1 v_1 (l_{i_t})^{v_1-1} + \psi_2 v_2 (l_{f_t})^{v_2-1}}{(1-\tau-\frac{1}{\mu})} \tag{25}$$

The first part of Equation 25 signifies the relative demand of formal and informal labour with

[18]The informal sector is known for low wages. However, in this paper, we are looking at informal labourers in formal employment. Recall, in Section 1 we talk of a new type of informal sector in India where informalisation occurs within the formal sector itself. Following Abraham (2017), informal employment in the formal sector in India includes a mix of blue and white-collar workers, and much like informal labourers in informal employment, there are construction workers amongst the informal labourers in the formal sector, as well as occupational profiles observed among the formally employed. Thus there exists a possibility informal wage can be higher than the formal wage. Similarly, although it may seem counter-intuitive that formal labourers are willing to become informal labourers in our model, A. Sinha (2010) empirically shows, using data from National Sample Survey of India (NSS) and Reserve Bank of India (RBI) data, in India formal labourers transform to informal labourers in face of rigidities.

[19]We assume the sum of the proportion of formal and informal labour adds up to a constant. Explanation given in the paragraph above Equation 20.

respect to the amount firms spend on formal wages compared to informal wages. The second part of Equation 25 signifies the relative supply for formal and informal labour with respect to the net amount formal labour receives compared to informal labour. Equating these two gives us the labour market equilibrium.

We obtain the first-order condition for capital from the differentiation of the profit equation (Equation 9) with respect to capital and equalising it to zero (details in Appendix A.2).

$$A_{f_t}(1-\alpha)k_{t-1}^{-\alpha}[l_{f_t}]^{\alpha} = r_t \tag{26}$$

where α is the formal labour share. Equation 26 shows that marginal product with respect to capital equates to the rental price of capital. Recall, the informal sector does not use capital (details in Section 2.2).

We differentiate the Lagrangian (Equation 96) with respect to capital and equate the resulting condition to zero to get equation 27.

$$R_{t-1} = \frac{r_t - 1 + \delta + \frac{1}{(k_{t-1})}(k_t - k_{t-1}) + \frac{1}{2(k_{t-1}^2)}(k_t - k_{t-1})^2}{1 + \frac{1}{(k_{t-2})}(k_{t-1} - k_{t-2})} \tag{27}$$

where δ is the depreciating factor. We equate Equation 27 with Equation 26 to obtain the equilibrium condition for capital, given by:

$$R_{t-1} = \frac{A_{f_t}(1-\alpha)k_{t-1}^{-\alpha}[l_{f_t}]^{\alpha} - 1 + \delta + \frac{1}{(k_{t-1})}(k_t - k_{t-1}) + \frac{1}{2(k_{t-1}^2)}(k_t - k_{t-1})^2}{1 + \frac{1}{(k_{t-2})}(k_{t-1} - k_{t-2})} \tag{28}$$

We differentiate the Lagrangian of the GHH utility function with respect to consumption and bonds to get the first-order condition for formal consumption, given by:

$$\frac{[c_{f_{t+1}} + c_{i_{t+1}} - \psi_1 l_{i_{t+1}}^{v_1} - \psi_2 l_{f_{t+1}}^{v_2}]^{-\sigma}}{[c_{f_t} + c_{i_t} - \psi_1 l_{i_t}^{v_1} - \psi_2 l_{f_t}^{v_2}]^{-\sigma}} = \frac{1}{R_t} \tag{29}$$

where σ shows the coefficient of risk aversion, ψ_1 is the informal labour weight in utility function, ψ_2 is the formal labour weight in utility function, v_1 is the intertemporal elasticity of informal labour supply and v_2 is the intertemporal elasticity of formal labour supply.

4.2 Log Linearised Solution

In this subsection, we provide the log linearised solution of the model. The step-by-step calculations are given in Appendix B1.

We log linearise Equation 25 to get Equation 30, the log linearised solution of the formal and informal labour market. Following the intuition of Marjit, Bhattacharyya, and L. Yang (2023) and Pappa, Sajedi, and Vella (2015), we are considering the proportion of formal and informal labour to be equal to one (refer to footnote 14 for details). We obtain Equation 31 from this condition.

$$\begin{aligned}
&\frac{\tilde{A}_f \alpha \tilde{k}^{1-\alpha} \tilde{l}_f^{\alpha-1}}{(1-\theta(1-\bar{R})-\frac{1}{\mu})\tilde{w}_f} \hat{A}_{f_t} - \frac{\tilde{A}_i \beta \tilde{l}_i^{-\beta-1}}{(1-\theta(1-\bar{R})-\frac{1}{\mu})\tilde{w}_f} \hat{A}_{i_t} \\
&+ \frac{\tilde{A}_f \alpha(\alpha-1)\tilde{k}^{1-\alpha}\tilde{l}_f^{\alpha-2} + \tilde{A}_i \beta(\beta-1)\tilde{l}_i^{\beta-2} k^{1-\beta}}{(1-\theta(1-\bar{R})-\frac{1}{\mu})\tilde{w}_f} \hat{l}_{f_t} \\
&= \frac{\theta \hat{R}_{t-1}}{1-\theta(1-\bar{R})-\frac{1}{\mu}} + \frac{\psi_2 v_2(v_2-1)\tilde{l}_f^{v_2-2} + \psi_1 v_1(v_1-1)\tilde{l}_i^{v_1-2}}{(1-\tau-\frac{1}{\mu})\tilde{w}_f} \hat{l}_{f_t}
\end{aligned} \tag{30}$$

$$\hat{l}_{ft} = -\hat{l}_{it} \tag{31}$$

The effect of real interest rate shocks and productivity shocks on formal and informal labour will depend on the relative demand and supply of formal and informal labour. The impact of the shocks on the labour market will further depend on the working capital parameter θ, the formal labour share in production α, the informal labour share in production β, the formal labour weight in utility function ψ_2, the informal labour weight in utility function ψ_1, the inter-temporal elasticity of informal labour supply ν_1, the inter-temporal elasticity of formal labour supply ν_2, the wage differential parameter μ. The impact of shocks on formal and informal further depends on the steady-state values of formal capital $\tilde{k}$, formal productivity level $\tilde{A}_f$, informal productivity level $\tilde{A}_i$, real interest rate $\tilde{R}$, formal labour $\tilde{l}_f$, informal labour $\tilde{l}_i$ and formal wage rate $\tilde{w}_f$.

Let us consider a scenario where $[1+\theta(\bar{R}-1)-\frac{1}{\mu}] > 0$ and $1-\tau-\frac{1}{\mu} > 0$. $1-\tau-\frac{1}{\mu} > 0$ implies the taxed formal wage is higher than the informal wage (refer to Section 4.1, especially the explanation in the paragraph following Equation 24). Following Equation 24, this means households are supplying formal labour more than informal labour. $[1+\theta(\bar{R}-1)-\frac{1}{\mu}] > 0$ implies the firm is willing to pay a higher amount for formal labour compared to informal labour (refer to Section 4.1, especially the explanation in the paragraph following Equation 22). Following Equation 22, we can say the relative demand for formal labour is higher than the relative demand for informal labour in this scenario.

In such a scenario, real interest rate shocks have a negative impact on formal employment and lead to a rise in informal employment. If the real interest rate rises it is now costlier for firms to pay formal wages; thus the firm will demand more informal labour. Positive formal productivity

shocks cause formal labour to rise and informal labour to fall. Positive informal productive shocks cause informal labour to rise and formal labour to fall.

The scenario $1-\tau-\frac{1}{\mu}<0$ and $[1+\theta(\bar{R}-1)-\frac{1}{\mu}]<0$ can be interpreted as that taxed formal wage is lower than informal wage and the firm is willing to spend less on formal labour as compared to informal labour. This means the relative demand and supply for informal labour is higher than that of formal labour. Even in that case, real interest rate shocks will lead to a rise in informal employment and a fall in formal employment. Formal productivity shocks will lead to a rise in formal employment and a fall in informal employment and vice versa for informal productivity.

Suppose $[1+\theta(\bar{R}-1)-\frac{1}{\mu}]<0$ and $1-\tau-\frac{1}{\mu}>0$. The relative demand for formal labour is lower than for informal labour. The relative supply of formal labour is higher than informal labour. This basically means, $1-\tau>\frac{1}{\mu}>1+\theta(\bar{R}-1)$. That cannot be captured in this model, as by definition of this model $1+\theta(\bar{R}-1)>1$ and τ is a fraction.

Let us consider that $[1+\theta(\bar{R}-1)-\frac{1}{\mu}]>0$ and $1-\tau-\frac{1}{\mu}<0$. The relative demand for formal labour is higher than informal labour. The relative supply of formal labour is lower than informal labour. There may exist a certain value of μ for which it is possible that $1-\tau<\frac{1}{\mu}<1+\theta(\bar{R}-1)$. Under the condition that the demand for formal labourers is higher than the supply of informal labourers, the formal productivity shocks lead to a rise in formal employment and a fall in informal employment, and the informal productivity shocks lead to a rise in informal employment and fall in formal employment, while real interest rate shocks lead to shift of labour a fall in formal employment and rise in informal employment.

We log linearise Equations 7 and 8 to get Equations 32 and 33, the log linearised solution of formal and informal output.

$$\hat{y}_{f_t}=\hat{A}_{f_t}+(1-\alpha)\hat{k}_{t-1}+\alpha\hat{l}_{f_t} \tag{32}$$

$$\hat{y}_{i_t}=\hat{A}_{i_t}+\beta\hat{l}_{i_t} \tag{33}$$

Formal output is impacted by formal productivity shocks directly. Real interest rate shocks and informal productivity shocks impact formal output through changes in formal labour and capital. The impact of shocks on formal output depends on the formal labour share and capital share of production.

Informal output is impacted by informal productivity shocks directly. Real interest rate shocks and formal productivity shocks impact informal output via changes in informal labour. The impact of shocks on formal output depends on the informal labour share of production.

We log linearise Equation 19 to get Equation 34, the log linearised solution of informal consumption. We log linearise Equation 29 to get Equation 35, the log linearised solution of formal consumption.

$$\hat{y}_{i_t} = \hat{c}_{i_t} \tag{34}$$

$$(\hat{c}_{f_{t+1}} - \hat{c}_{f_t})\tilde{c}_f + (\hat{y}_{i_{t+1}} - \hat{y}_{i_t})\tilde{c}_i = \frac{1}{\sigma}(\tilde{c}_f + \tilde{c}_i - \psi_1 \tilde{l}_i^{v_1} - \psi_2 \tilde{l}_f^{v_2})\hat{R}_t + (1 - \tau - \frac{1}{\mu})\tilde{w}_f(\hat{l}_{f_{t+1}} - \hat{l}_{f_t}) \tag{35}$$

The impact of shocks on informal consumption depends on the impact of shocks on informal output. Real interest rate shocks impact formal consumption paths directly as well as indirectly through formal labour. The impact of shocks on formal consumption depends on the parameters σ which is the coefficient of risk aversion, ψ_1 which is the informal labour weight in the utility function, ψ_2 which is the formal labour weight in the utility function, v_1 which is the intertemporal elasticity of informal labour supply, v_2 which is the intertemporal elasticity of formal labour supply. The impact of shocks on formal consumption further depends on the steady-state values of formal consumption $\tilde{c}_f$, informal consumption $\tilde{c}_i$, formal labour $\tilde{l}_f$, informal labour $\tilde{l}_i$ and formal wage rate $\tilde{w}_f$.

We log linearise Equation 28 to get Equation 36, the log linearised solution of formal capital.

$$\begin{aligned} &-\frac{\frac{(1-\alpha)\tilde{y}_f}{\tilde{k}}}{\frac{(1-\alpha)\tilde{y}_f}{\tilde{k}} - 1 + \delta}\hat{A}_{f_t} + \hat{R}_{t-1} + [\hat{k}_{t-1} - \hat{k}_{t-2}] \\ &= \frac{(1-\alpha)(-\alpha)\frac{\tilde{y}_f}{\tilde{k}}}{(1-\alpha)\frac{\tilde{y}_f}{\tilde{k}} + (1-\delta)}\hat{k}_{t-1} - \frac{(1-\alpha)(-\alpha)\frac{\tilde{y}_f}{\tilde{k}}}{(1-\alpha)\frac{\tilde{y}_f}{\tilde{k}} + (1-\delta)}\hat{l}_{f_t} + \frac{[\hat{k}_t - \hat{k}_{t-1}]}{(1-\alpha)\frac{\tilde{y}_f}{\tilde{k}} + (1-\delta)} \end{aligned} \tag{36}$$

The effect of the shocks on capital depends on the values of formal labour exponent α, steady state formal output to capital ratio $\frac{\tilde{y}_f}{\tilde{k}}$and depreciation rate δ.

We log linearise Equation 3 to get Equation 37, the log linearised solution of investment.

$$\hat{x}_t = \frac{\tilde{k}\hat{k}_t - (1-\delta)\tilde{k}\hat{k}_{t-1}}{\delta k} \tag{37}$$

The effect of the shocks on investment depends on the steady-state level of capital $\tilde{k}$ and depreciation rate δ.

We log linearise Equation 10 to get Equation 38, the log linearised solution of government expenditure.

$$\hat{g}_t = \frac{\psi_2 v_2(v_2 - 1)\tilde{l}_f^{v_2 - 2} + \psi_1 v_1(v_1 - 1)(\tilde{l}_i)^{v_1 - 2}}{1 - \tau - \frac{1}{\mu}}\tilde{l}_f\hat{l}_{f_t} + \hat{l}_{f_t} \tag{38}$$

Changes in government expenditure are dependent on the tax rate τ and the wage differential between formal and informal labour μ.

We log linearise Equation 18 to get Equation 39, the log linearised solution of net exports.

$$\hat{nx}_t = \frac{\tilde{y}_f}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{g}}\hat{y}_{f_t} - \frac{\tilde{c}_f}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{g}}\hat{c}_{f_t} - \frac{\tilde{x}}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{g}}\hat{x}_t - \frac{\tilde{g}}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{g}}\hat{g}_t \quad (39)$$

The impact of shocks on net export depends on the steady-state values of formal output $\tilde{y}_f$, formal consumption $\tilde{c}_f$, investment $\tilde{x}$ and government expenditure $\tilde{g}$.

5 Calibration

In this section, we discuss the calibration of the DSGE model for India with data from 1996 to 2016. The calibrated values have been put forward in Table 2.

In the upper panel of Table 2, we show the name of the parameters, the symbol of the parameters that have been used in our DSGE model, followed by the values. Then we put the source of the calibrated values. If the values are calculated using the model, we denote the source by 'Calculated from Model'. If the values are obtained from data, we denote the source by 'Data-driven'. Otherwise, we just cite the paper where the value has been obtained. We then describe them in detail in the following paragraphs.

In this paragraph, we discuss the parameters sourced from the literature. We follow Neumeyer and Perri (2005) and Tiryaki (2012) to set the formal labour weight under GHH preferences to be 1.6. We could not find the value of the informal labour weight for India anywhere. Following Fajnzylber (2001) and Gindling and Terrell (2005), we find that for Brazil, the inverse of wage elasticity of informal labour supply[20] is 1.03 for informalised labour in the formal labour market (the type of informal sector we are considering in our analysis) and 1.32 for self-employed informal labour. Due to the lack of literature or data for India, we take the wage elasticity of the informal labour supply of Brazil as a proxy for the wage elasticity of the informal labour supply for India. Following Neumeyer and Perri (2005), the labour curvature is given by the sum of the inverse of wage elasticity and one. Thus, we obtain the value of 2.03 for informal labour curvature in India. The wage differential parameter[21] for formal employment and informal employment is calculated from Singhari and Madheswaran (2017), M. Kumar and S. Pandey (2021) and Narayanan (2015)[22].

[20]The inverse of wage elasticity represents how much wages need to change to elicit unit change in labour supply, emphasising the rigidity or responsiveness of labour supply to wage changes.

[21]We calculate that the wage differential is 2.65 when we consider informal and formal interlock of formal employment. However, for the whole of the informal to formal sector in India, the wage rate is

[22]All three of them have reported formal and informal wage using National Sample Survey data on Employment

We obtain the ratio of time devoted to informal labour in formal employment from Abraham (2017) and Raveendran and Vanek (2020). We find that the majority of informal labourers who are employed in the formal sector work 53 hours per week compared to the mandated 40 hours of formal workers, following the Minimum Wage Act. The income tax rate in India is 5%, 10%, 15% and 20% for different levels of income. We will consider the tax rate to be 10%[23] for our benchmark analysis. From the DGE estimates of the ratio of informal output compared to total output by Elgin et al. (2021), we get that formal output in India represents 80% of the economy compared to 20% of the informal economy. We obtain the value of risk aversion from S. Sinha and Kamaiah (2017)[24].

In this paragraph, we describe the calibrated values that have been calculated from the data. The real interest rate $\bar{R}$ is calculated on the quarterly inflation-adjusted call money rate (data from Federal Reserve Economic Data). In the last chapter, we conclude using data from World Bank the average wage bill in India is 54%. We deduce that this value is the average of the mean formal wage bill and mean informal wage ill. Following Equation 21, we calculate the values of formal and informal wage bills such that $\frac{\tilde{w}_f + \frac{\tilde{w}_f}{\mu}}{2} = 0.54$. The depreciation rate is obtained from the ratio of investment to output (Data from the Reserve Bank of India). The working capital θ is given by gross fixed capital formation of the private sector expressed as a percentage of GDP and is obtained from World Bank.

We further discuss the calibrated values obtained from the model. Following Neumeyer and Perri (2005) and Tiryaki (2012) and Gupta (2025), the capital exponent can be given by the share of the rental bill to formal output. Suppose we do not have informal production in our model. In the presence of only a formal sector, if we differentiate the profit equation with respect to capital and equal it to zero, we get $(1-\alpha)A_{f_t} l_{f_t}^{\alpha} k_{t-1}^{-\alpha} = r_t$. We can rewrite the equation such that, at steady state

$$(1-\alpha) = \frac{\tilde{r}\tilde{k}}{\tilde{y}_f} \tag{40}$$

and Unemployment. We calculate the wage differential by dividing the mean formal wage by the mean informal wage. Narayanan (2015) have used two definitions of informal-formal wage. Their second definition of the informal sector matches that of our definition which helps us extract the formal-informal wage differential.

[23]The 10% income tax rate in India is used in examples because it represents a significant bracket within the tax structure for middle-income earners. Taxpayers earning between Rs. 7 lakh and Rs. 10 lakh fall into this 10% category, which is a key demographic in terms of taxable income and financial policy impact. However, in 2023-24, around 70% of the individuals who filed returns had no tax liability, highlighting the narrow base of actual taxpayers in India. We reflect these statistics in our analysis by modelling workers who do not pay taxes as part of the informal sector. Among those who earn and pay tax rates, 10% represents the majority of them in India. We source these statistics from newspaper articles and technical reports (S. Sengupta 2024, Raj 2024, Das 2024).

[24]S. Sinha and Kamaiah (2017) compare the risk-neutral probability density's forecasting ability and risk-adjusted density functions to arrive at a unique index of relative risk aversion for Indian markets.

Parameter	Symbols	Value	Source
Real Interest Rate	$\bar{R}$	1.056	Data-driven (Federal Reserve Economic Data)
Formal Labour Exponent	α	0.64	Calculated From Model
Informal Labour Exponent	β	0.3	Calculated From Model
Ratio of time denoted to formal labour	$\tilde{l}_f$	0.239	Raveendran and Vanek (2020)
Ratio of time denoted to informal labour	$\tilde{l}_i$	0.31	Minimum Wage Act of India
Working Capital Parameter	θ	0.27	Data-driven (World Bank)
Coefficient of risk aversion	σ	8	S. Sinha and Kamaiah (2017)
Formal labour curvature	ν_2	1.6	Neumeyer and Perri (2005)
Informal labour curvature	ν_1	2.03	Gindling and Terrell (2005) and Fajnzylber (2001)
Formal wage bill	$\tilde{w}_f$	0.81	Calculated From Model + Data-driven (World Bank)
Informal wage bill	$\tilde{w}_i$	0.3	Calculated From Model + Data-driven (World Bank)
Wage differential parameter	μ	2.65	Narayanan (2015)
Formal labour weight	ψ_2	0.82	Calculated From Model
Informal labour weight	ψ_1	0.49	Calculated From Model
Steady state Formal Consumption to Output	$\tilde{c}_f$	0.78	Calculated From Model + Data-driven (Reserve Bank of India)
Steady state Informal Consumption to Output	$\tilde{c}_i$	1	Calculated From Model
Steady state Formal Output to Total Output	$\tilde{y}_f$	0.8	Elgin et al. (2021)
Steady state Informal Output to Total Output	$\tilde{y}_i$	0.2	Elgin et al. (2021)
Tax Rate	τ	0.10	S. Sengupta (2024), Raj (2024) and Das (2024)
Depreciation Rate	δ	0.031	Data-driven (Reserve Bank of India)

Shock Name	Parameter Value	Volatility Value	Shock Process
Formal productivity	ρ_{A_f}=0.48	σ_{A_f}= 0.06	$\hat{A}_{f_t} = \rho_{A_f}\hat{A}_{f_{t-1}} + \epsilon_{A_{f_t}}$
Informal productivity	ρ_{A_i}=0.21	σ_{A_i}= 0.01	$\hat{A}_{i_t} = \rho_{A_i}\hat{A}_{i_{t-1}} + \epsilon_{A_{i_t}}$
International Rate of Interest	ρ_{R^*}=0.72	σ_{R^*}= 0.52	$\hat{R}^*_t = \rho_{R^*}\hat{R}^*_{t-1} + \epsilon_{R^*_t}$
Country Risk (Independent)	ρ_D= 0.32	σ_D= 1.53	$\hat{D}_t = \rho_D\hat{D}_{t-1} + \epsilon_D$
Country Risk (Formal Productivity Induced)	η_f=-7.29	σ_{η_f}= 1.58	$\hat{D}_t = \tilde{\eta}_f(E_t\hat{A}_{f_{t+1}}) + \epsilon_{\eta_{f_t}}$
Country Risk (Informal Productivity Induced)	η_i=1.03	σ_{η_i}= 1.67	$\hat{D}_t = \tilde{\eta}_i(E_t\hat{A}_{i_{t+1}}) + \epsilon_{\eta_{i_t}}$

Table 2: Calibration

Our formal production function (Equation 7) follows constant returns to scale. The formal labour exponent of India is given by differencing the capital exponent from 1.

Suppose we do not have formal production in our model. In the presence of only the informal sector, we differentiate the profit equation with respect to informal labour. We get $\beta A_{i_t} l_{i_t}^{\beta-1} = w_{i_t}$. We can rewrite the equation such that at steady state:

$$\beta = \frac{\tilde{w}_i \tilde{l}_i}{\tilde{y}_i} \tag{41}$$

We deduce the value of the informal labour exponent from the informal wage bill.

Suppose we do not have a formal sector in our model. In the presence of only the informal sector, if we differentiate the Lagrangian equation with respect to informal labour and equate it to zero, we get the following:

$$\psi_1 \nu_1 \tilde{l}_i^{\nu_1 - 1} = \tilde{w}_i \tag{42}$$

We already know the steady state values of informal labour, informal wage and informal labour curvature. We thus calculate informal labour weight using Equation 42.

Similarly,

$$\psi_2 \nu_2 \tilde{l}_f^{\nu_2 - 2} = \tilde{w}_f \tag{43}$$

We thus calculate formal labour weight using Equation 43.

Following Equation 19, we obtain the consumption output ratio for informal output is 1. In Gupta (2025) the ratio of total consumption to total output in India is 0.7. Using that and the fact informal consumption is equal to informal output and the ratio of informal to formal output is $1/4$, we calculate the value of formal consumption to formal output ratio.

In the lower panel of Table 2, we put forward the shock processes as outlined in Section 3.4. Formal productivity, informal productivity and foreign real interest rate shocks follow AR(1) processes. Country risk shocks follow an AR(1) process in the independent country risk approach (refer to Section 3.4). We refer to the country risk process modelled as the independent country risk approach as 'Country Risk (Independent)' in Table 2. Country risk depends on formal productivity in the formal productivity-induced approach (refer to Section 3.4). We refer to country risk shocks modelled as formal productivity induced approach as 'Country Risk Formal Productivity Induced)' in Table 2. Country risk shocks depend on informal productivity in the informal productivity-induced approach (refer to Section 3.4). We refer to country risk shocks modelled as informal productivity induced approach as 'Country Risk Informal Productivity Induced) ' in Table 2. For both the induced cases, we have used OLS regressions to obtain the dependence of country risk on formal and informal productivity levels[25]. We put forward the processes these shocks follow on the far right of the lower panel of Table 2. We estimate these processes[26]. The heading 'Parameter Value' lists the estimated values of the persistence parameter for shocks following AR(1) processes and the estimated values of the dependence parameter for country risk shocks depending on levels of formal and informal productivity. The heading 'Volatility Value' lists the values of the estimated volatility of the processes.

Now, we discuss the parameter and volatility values obtained from estimating the mentioned shock processes, as shown in the lower panel of Table 2. Following parameter values, the effect of

[25]We use this method following Neumeyer and Perri (2005), who argue that although it is not the most robust method, it helps in capturing the degree of dependence which is what we really want.

[26]Following the last chapter, the foreign real interest rate is calculated by subtracting the US inflation rate (Federal Reserve Economic Data) from the effective funds rate of the US (Federal Reserve Economic Data). The real interest rate is calculated by subtracting the Indian inflation rate (Federal Reserve Economic Data) from the call money rate of India (Federal Reserve Economic Data). Following Neumeyer and Perri (2005) and Equation (3.13), country risk is calculated from the difference of the HP filtered values of Indian and US real interest rates. Deviation of formal and informal productivity from the steady state is calculated using Equations 32 and 33. This method is recommended by Tiryaki (2012) when data is unavailable. Data for the proportion of formal labour, informal labour, formal output and informal output has been calculated from Elgin et al. (2021). Details about data have been put in Section 8.1 and later in Appendix A.1.

foreign real interest rate shocks in India is persistent. Country risk is volatile, no matter the origin of the shocks[27]. Country risk is highly dependent on formal productivity. Country risk is more volatile when dependent on formal productivity (formal productivity induced approach) compared to independent causes[28]. Country risk is positively affected by the rise in informal productivity. A rise in informal productivity raises concerns that the informal sector's growth often occurs outside regulatory frameworks and raises concerns about the government's ability to enforce laws and ensure economic stability (Schneider and Enste 2000). A rise in informal productivity also signals an increase in poverty and income inequality (Unni and R. Naik 2013) and less access for labourers to welfare or representation (Harriss-White 2003). These indications increase investor concerns, which can explain why the rise in informal productivity in India increases country risk. We find that country risk is less dependent on informal productivity compared to formal productivity. An explanation for this can be the harder quantification (Vanek et al. 2014) and limited visibility (Nason et al. 2024) of the informal sector, especially when it is contained within the formal sector following our model[29]. Formal productivity is more persistent in the economy compared to informal productivity. Formal productivity is more volatile than informal productivity in India. Both Leyva and Urrutia (2020) and Coşkun (2022) find that the volatility of formal productivity is higher than informal productivity for Mexico. An explanation could be informal labourers may have more flexibility to adjust in response to the informal productivity shocks (Alcaraz 2009, Yadav 2021) mitigating the overall volatility of informal productivity. However, country risk borne out of informal productivity is more volatile than country risk borne out of formal productivity.

6 Simulated Impulse Response Functions

In this section, we generate the simulated impulse responses for the scenarios of foreign real interest rate shocks, country risk shocks (three different origins of country risk shocks: independent causes, formal productivity induced and informal productivity induced), formal productivity shocks (both scenarios when formal productivity shocks induce or do not induce country risk shocks) and informal productivity shocks (both scenarios when informal productivity shocks induce or do not induce country risk shocks)[30]. The standard deviation of shocks for all simulated impulse responses in this paper is one.

[27]Recall, country risk shocks can be caused in three ways: AR(1) process, induced by formal productivity and induced by informal productivity. Refer to Section 3.4 and Table 1 for details.

[28]An interpretation can be that uncertainty surrounding India's formal productivity amplifies investor concerns, leading to heightened volatility in country risk metrics.

[29]Recall, we look into informalisation of the formal sector.

[30]Refer to Table 1 for detailed description of the shocks and their expected impact. Refer to Section 4.4 for detailed modelling of the shocks.

6.1 Foreign Real Interest Rate Shocks

In this subsection, we examine the impact of foreign interest rate shocks on the Indian formal and informal economy. We refer to Figure 1 and Figure 2.

Due to shocks to the foreign real interest rate, the Indian real interest rate rises. Recall that firms have a contract with formal labour. Firms need to pay formal labour a portion of their wages before production has taken place. Firms take out a working capital loan. Real interest rate rises. The amount for a working capital loan is now higher. Formal employment falls. Informal labour has no such contract with firms. Firms thus have higher informal labour. Informal employment rises. Since the real interest rate is affecting the economy with a lag (refer to Equation 30), over the first two periods, the effect of the foreign real interest rate shocks on the economy worsens and then gradually wears off. Our results match with the intuition of Marjit, Bhattacharyya, and L. Yang (2023), who use a general equilibrium model to show that in the face of demand shocks, expansion of formal employment leads to contraction of informal employment.

When the real interest rate rises, it is harder for firms to take out working capital loans and as a result, firms are less likely to invest. This follows conventional economic theory, where various papers have explained this using different models (for example McKenna and Zannoni 1990).

When there are foreign real interest rate shocks, informal output rises because of the rise in informal labour (recall Equation 33). Informal consumption rises along with the rise in informal output (recall Equation 34). Formal consumption falls at first but then steadily increases. This can be attributed to the Euler equation, which states that with the rise in real interest rate, current consumption falls in favour of future consumption. Thus, at first, current formal consumption decreases because individuals save, but over further periods, individuals start spending their savings, so future consumption increases. Net exports are affected positively when the total spending is less than the total output produced because of the fall in consumption, investment and government expenditure. However, the rise in future consumption leads to more spending than production, thus resulting in a fall in net exports in future (recall Equation 3.9). Due to foreign real interest rate shocks, formal labour decreases, fewer individuals pay taxes, government revenue falls and in the face of the balanced budget condition, government expenditure falls.

6.2 Country Risk Shocks

In this subsection, we examine the impact of country risk shocks on the Indian formal and informal economy. If we recall from Section 3.4 and Table 1, country risk shocks have been modelled in three ways.

We first refer to the independent country risk approach, where country risk shocks are borne

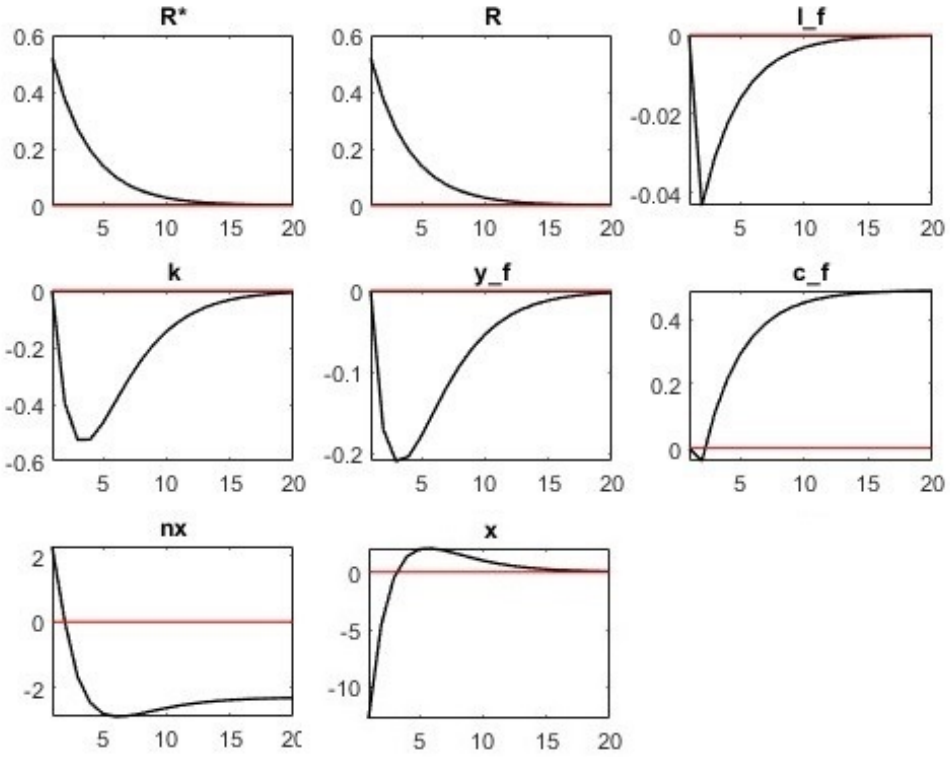

Figure 1: Effect of Foreign Real Interest Rate Shocks On Formal Economy

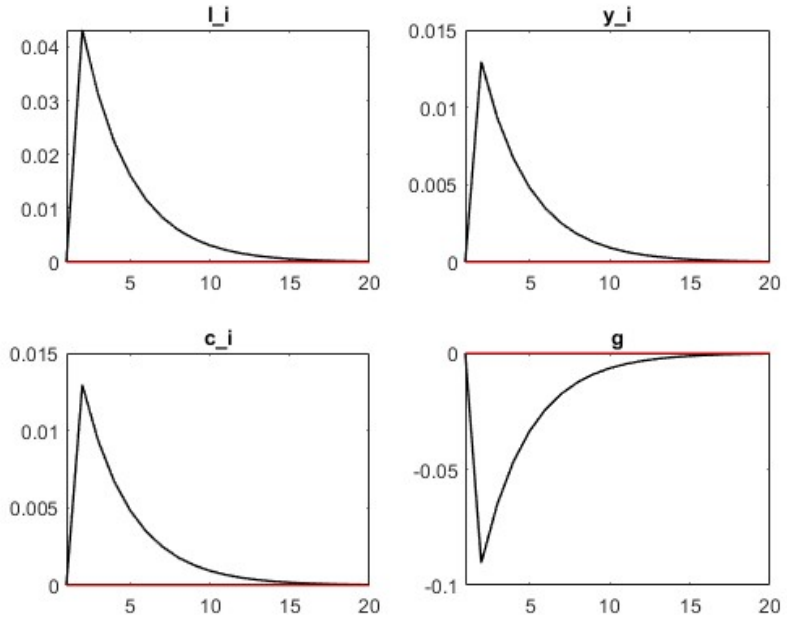

Figure 2: Effect of Foreign Real Interest Rate Shocks On Informal Economy and Government Expenditure

R^* refers to foreign interest rate, R refers to real interest interest rate, l_f refers to formal labour, k stands for capital,y_f refers to formal output, c_f refers to formal consumption, nx refers to net exports, x refers to investment,l_i refers to informal labour, y_i refers to informal output, c_i refers to informal consumption and g refers to government expenditure.

of independent causes and are not induced by any fundamental shocks in the economy (refer to Equation 15). Figures 3 and 4 show the impact of country risk shocks on the Indian formal and informal economy when the country risk shocks are modelled using the independent country risk approach.

Secondly, we refer to the formed productivity-induced approach, where country risk shocks are induced by formal productivity shocks (refer to Equation 16). When formal productivity rises, it causes country risk to fall (refer to Table 2). Figures 5 and 6 show the impact of country risk shocks on the Indian formal and informal economy when the country risk shocks are modelled using the formal productivity-induced approach.

Thirdly, we refer to the informed productivity-induced approach, where country risk shocks are induced by formal productivity shocks (refer to Equation 16). When informal productivity rises, it causes country risk to rise (refer to Table 2). Figures 7 and 8 show the impact of country risk shocks on the Indian formal and informal economy when the country risk shocks are modelled using the informal productivity-induced approach.

Irrespective of the cause of the country risk shocks, in all three scenarios, the signs of the effect of the shocks on the formal and informal economy remain the same. If country risk increases, real interest rate rises, investment is likely to fall fast. When country risk shocks occur, the real interest rate increases. In that case, it becomes costlier to hire formal labour as it is harder to get the working capital loan, given firms have to pay a portion of the firm's wages before production. Firms hire informal labour instead. Given no change in labour supply, formal employment falls, and informal employment decreases. Capital falls. The fall in capital and formal labour causes formal output to fall. The rise in the proportion of informal labour due to country risk shocks translates to lesser tax revenue for the government. Given the balanced budget condition, the fall in government revenue translates into a fall in government expenditure.

A rise in informal labour resulting from country risk shocks will result in a rise in informal output (recall Equation 33) and, consequently, informal consumption[31] (recall Equation 34). Initially, a rise in the real interest rate causes a decline in current formal consumption as individuals tend to save more for future consumption. However, over subsequent periods, formal consumption gradually increases. This pattern can be explained by the Euler equation, which suggests that as the real interest rate increases, individuals allocate less to current consumption in favour of future consumption. Net exports are positively affected when total spending is less than total output produced, driven by decreases in consumption, investment, and government expenditure (recall Equation 3.9). Nevertheless, as future consumption rises, leading to consumption exceeding

[31]The positive link between country risk and informality has been shown empirically in the literature on developing countries (Dohmen, Khamis, Lehmann, et al. 2010, N. Loayza et al. 2007).

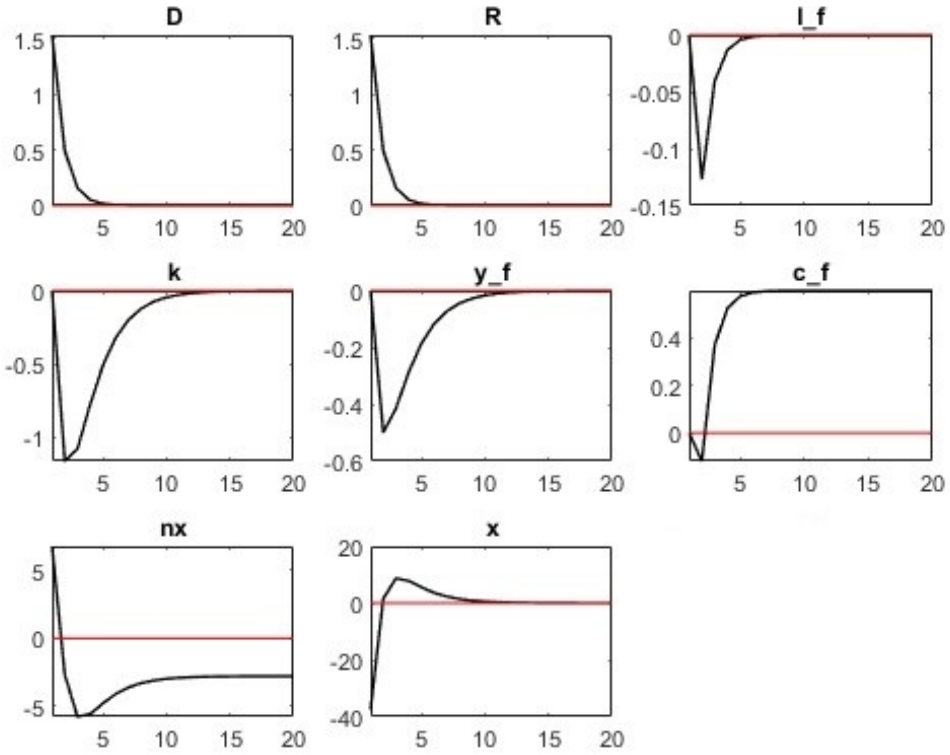

Figure 3: Effect of Country Risk Shocks (Independent Case) On Formal Economy

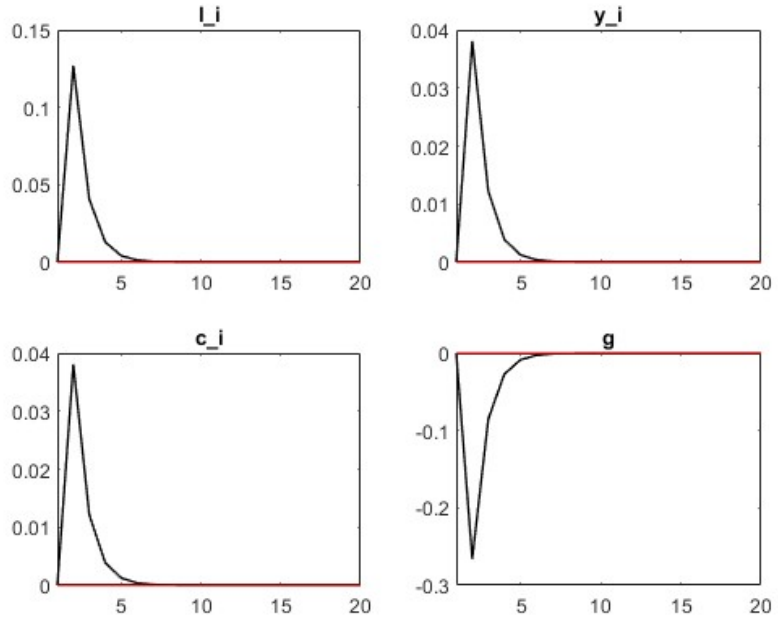

Figure 4: Effect of Country Risk Shocks (Independent Case) On Informal Economy and Government Expenditure

D refers to country risk, *R* refers to real interest rate, l_f refers to formal labour, k stands for capital,y_f refers to formal output, c_f refers to formal consumption, *nx* refers to net exports, *x* refers to investment,l_i refers to informal labour, y_i refers to informal output, c_i refers to informal consumption and *g* refers to government expenditure.

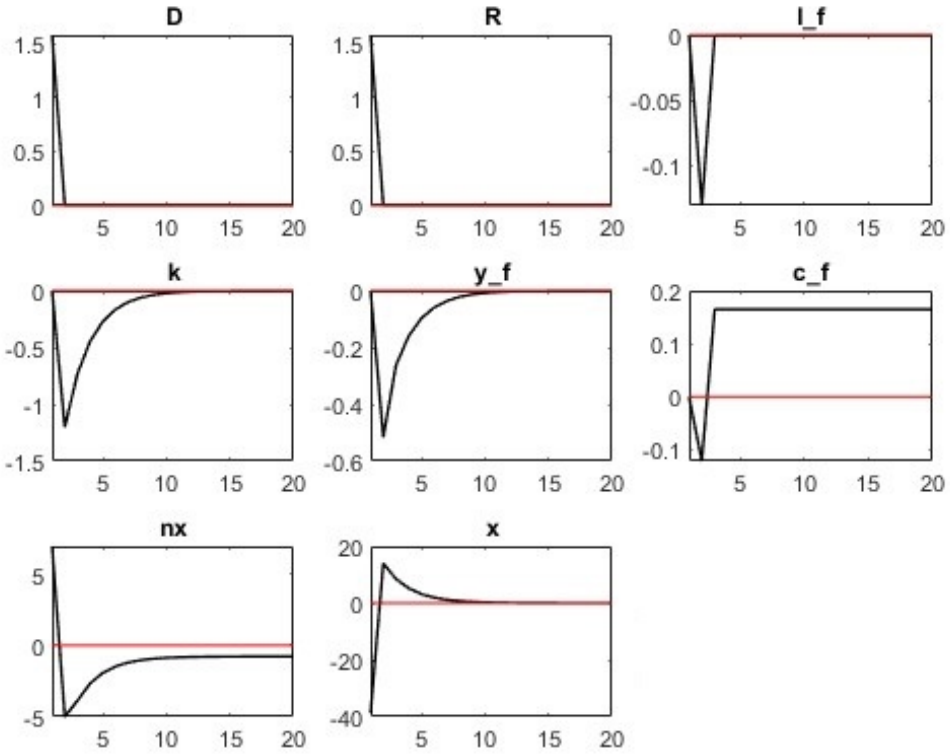

Figure 5: Effect of Country Risk Shocks (Formal Productivity Induced Case) On Formal Economy

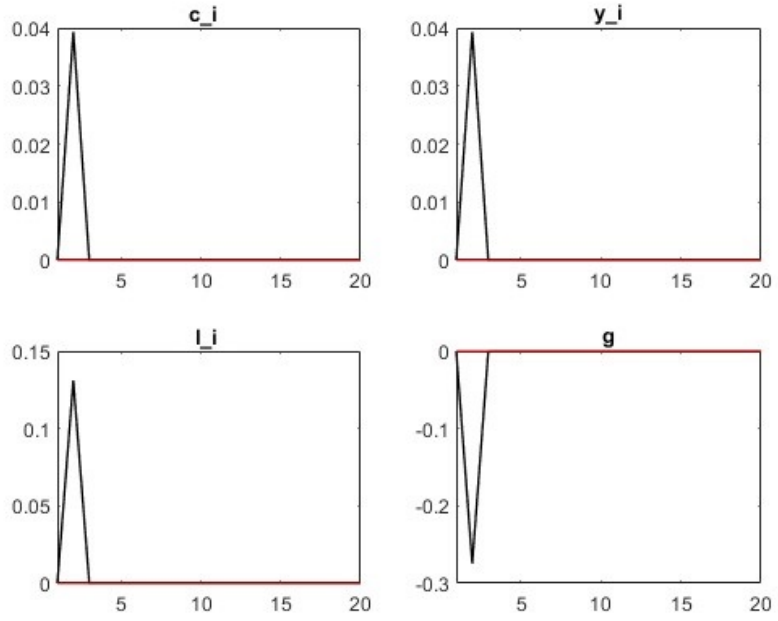

Figure 6: Effect of Country Risk Shocks (Formal Productivity Induced Case) On Informal Economy and Government Expenditure

D refers to country risk, R refers to real interest rate, l_f refers to formal labour, k stands for capital,y_f refers to formal output, c_f refers to formal consumption, nx refers to net exports, x refers to investment,l_i refers to informal labour, y_i refers to informal output, c_i refers to informal consumption and g refers to government expenditure.

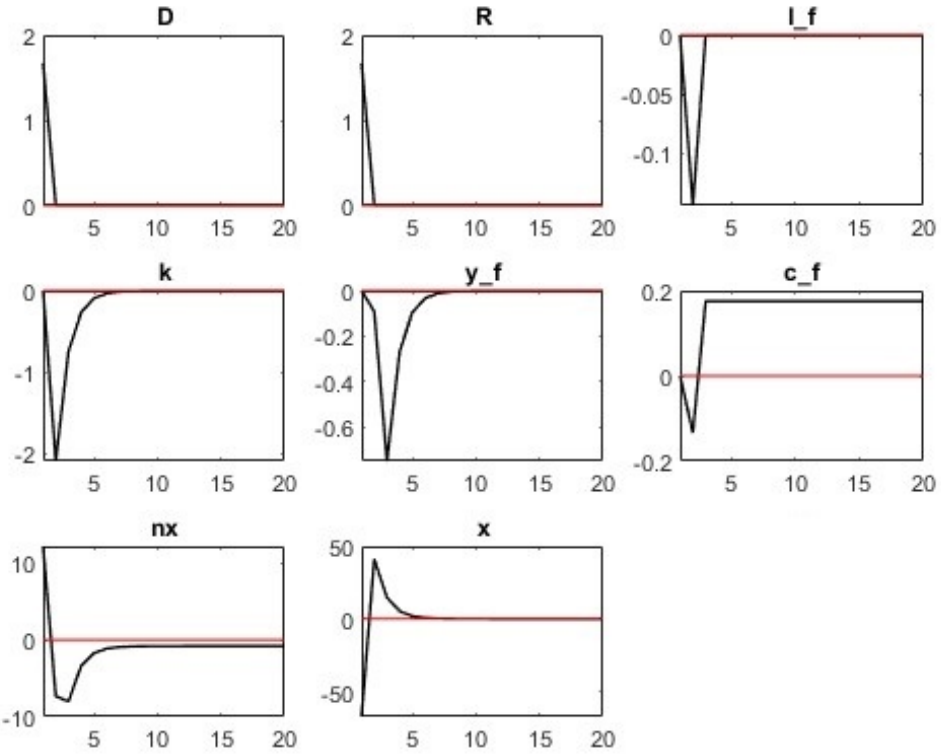

Figure 7: Effect of Country Risk Shocks (Informal Productivity Induced Case) On Formal Economy

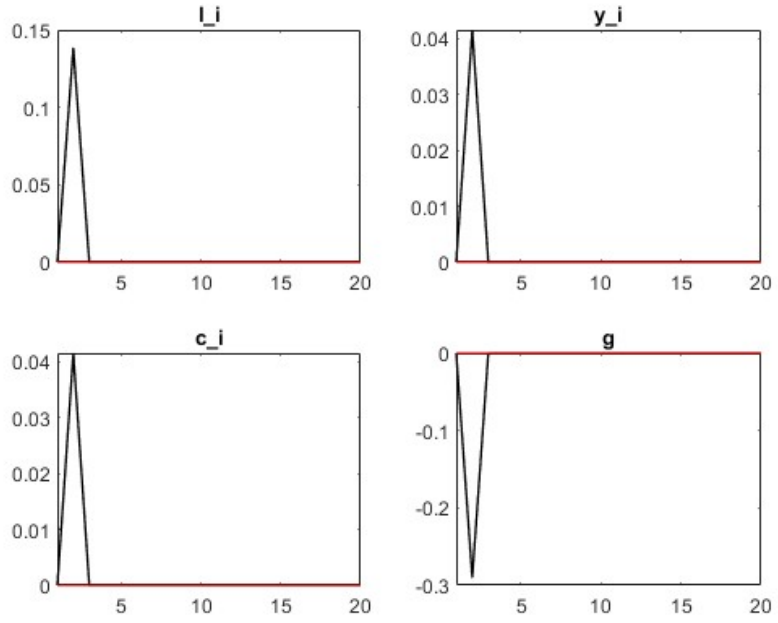

Figure 8: Effect of Country Risk Shocks (Informal Productivity Induced Case) On Informal Economy and Government Expenditure

D refers to country risk, R refers to real interest rate, l_f refers to formal labour, k stands for capital,y_f refers to formal output, c_f refers to formal consumption, nx refers to net exports, x refers to investment,l_i refers to informal labour, y_i refers to informal output, c_i refers to informal consumption and g refers to government expenditure.

production, net exports decline.

Even though the sign of the effect of the shocks is the same regardless of the origin of country risk shocks, the persistence differs. Country risk shocks, when influenced by formal productivity shocks or informal productivity shocks, are more pronounced with more kinks and are very similar in impact.

In the case where country risk shocks follow a simple AR(1) process, as in the independent country risk approach, the shocks evolve in a smooth, autoregressive manner, primarily driven by their own past values. This results in a more stable and predictable adjustment over time. In contrast, in the formal and informal induced scenario, country risk shocks are dependent on external factors, which makes the impact of these shocks more abrupt and pronounced.

6.3 Formal Productivity Shocks

In this subsection, we examine the impact of formal productivity shocks on the Indian formal and informal economy. We see both scenarios where formal productivity shocks do not have any impact on the economy and when formal productivity shocks induce country risk shocks. Figures 9 and 10 represent the impact of formal productivity shocks on the Indian formal and informal economy, in the scenario formal productivity shocks have no impact on country risk shocks. Figures 11 and 12 represent the impact of formal productivity shocks on the Indian formal and informal economy, in the scenario formal productivity shocks negatively induce country risk shocks.

Positive formal productivity shocks lead to a rise in formal employment in India. Capital stock and investment increase. This leads to a rise in formal output, which also translates to a rise in formal consumption. Government revenue increases since formal employment increases. Informal employment falls, and as such, informal output and consumption decreases. Our results are consistent with findings reported in previous literature such as Mueller (1972), Chong, Galdo, and Saavedra (2008), C. Elgin and Birinci (2016), Pradeep, M. Bhattacharya, and Chen (2017), Horvath (2018), Leyva and Urrutia (2020) and Opondo, Etyang, and Ayieko (2022).

In the induced case, the effects of the formal productivity shocks are much more pronounced than in the independent case. When formal productivity increases, country risk falls, and subsequently, the real interest rate decreases. There are dual impacts on both formal and informal economy; both the rise in productivity and the subsequent fall in country risk shocks impact the Indian formal and informal economy. When country risk falls, the real interest rate falls, working capital loans become cheaper, and firms choose to employ formal labour compared to informal labour. Formal output increase. Informal output falls. Thus the impact of productivity shocks on both formal and informal economy are amplified when formal productivity shocks depend on country risk shocks.

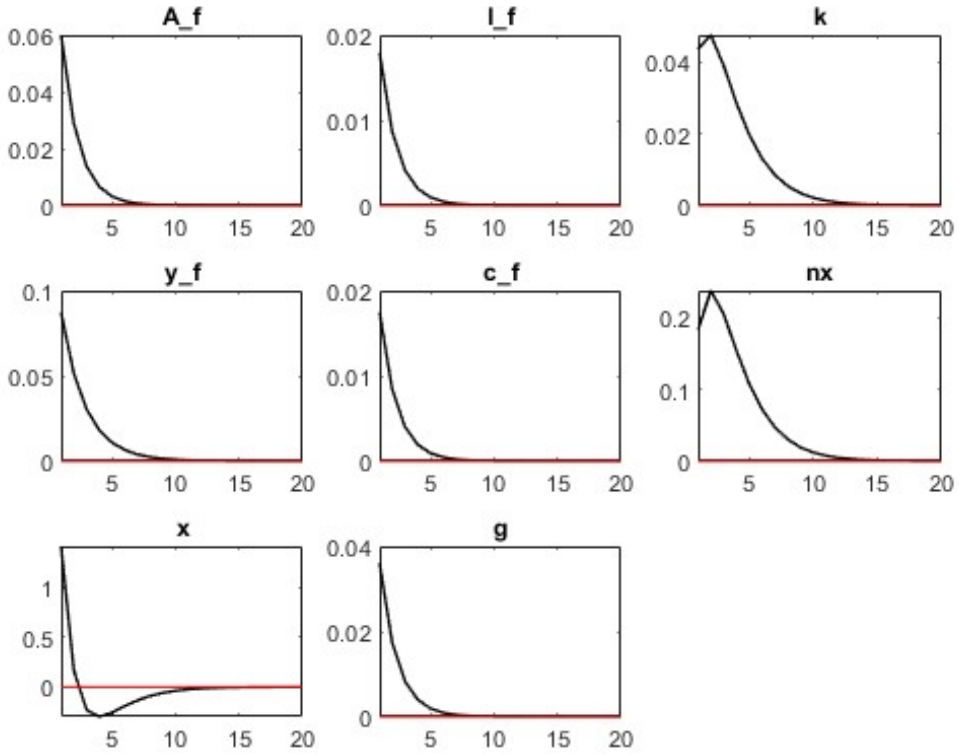

Figure 9: Effect of Formal Productivity Shocks (Independent Case) On Formal Economy

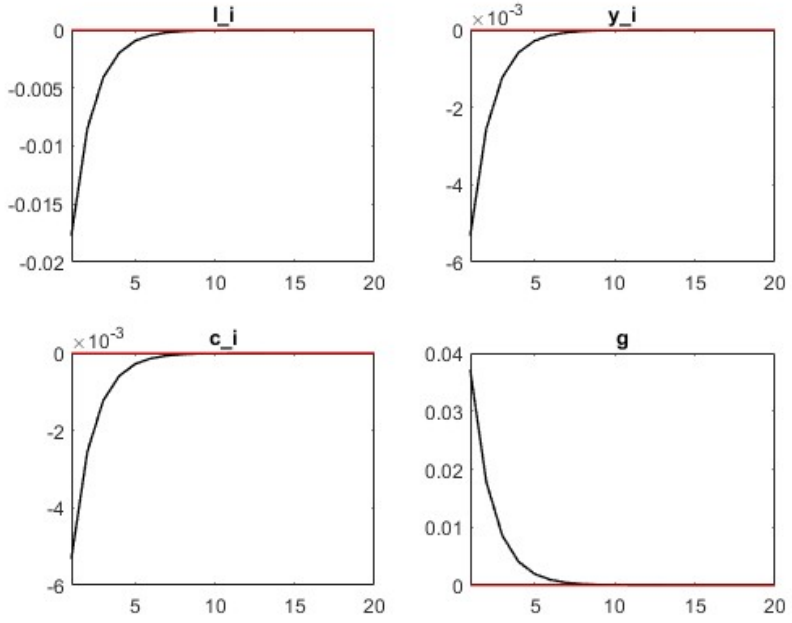

Figure 10: Effect of Formal Productivity Shocks (Independent Case) On Informal Economy and Government Expenditure

A_f refers to formal productivity, R refers to real interest rate, l_f refers to formal labour, k stands for capital,y_f refers to formal output, c_f refers to formal consumption, nx refers to net exports, x refers to investment,l_i refers to informal labour, y_i refers to informal output, c_i refers to informal consumption and g refers to government expenditure.

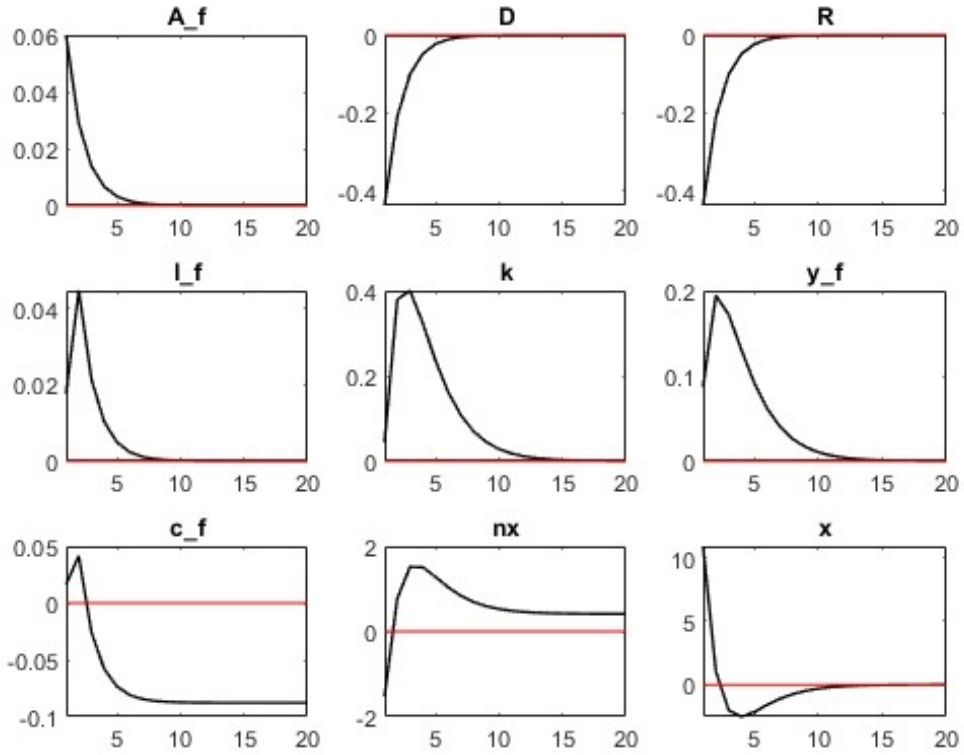

Figure 11: Effect of Formal Productivity Shocks (Formal Productivity Induced Case) On Formal Economy

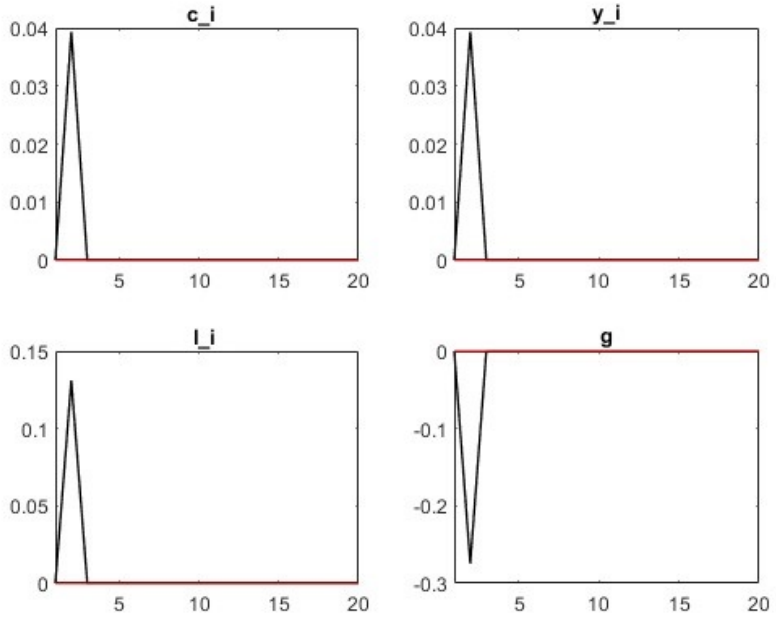

Figure 12: Effect of Formal Productivity Shocks (Formal Productivity Induced Case) On Informal Economy and Government Expenditure

A_f refers to formal productivity, D refers to country risk, R refers to real interest rate, l_f refers to formal labour, k stands for capital,y_f refers to formal output, c_f refers to formal consumption, nx refers to net exports, x refers to investment,l_i refers to informal labour, y_i refers to informal output, c_i refers to informal consumption and g refers to government expenditure.

6.4 Informal Productivity Shocks

In this subsection, we examine the impact of informal productivity shocks on the Indian formal and informal economy. We see both scenarios where informal productivity shocks do not have any impact on the economy and when informal productivity shocks induce country risk shocks. Figures 13 and 14 represent the impact of formal productivity shocks on the Indian formal and informal economy, in the scenario formal productivity shocks have no impact on country risk shocks. Figures 15 and 16 represent the impact of formal productivity shocks on the Indian formal and informal economy, in the scenario formal productivity shocks negatively induce country risk shocks.

Informal productivity shocks lead to a rise in informal employment in India. As a result, informal output and informal consumption increase. However, there is a decrease in formal employment. Capital stock, being complementary to formal labour, decreases, and so does investment. Formal consumption decreases, given the fall in formal output. A fall in formal employment means less tax revenue for the government, hence government revenue decreases. There is a scarcity of literature specifically examining the impact of informal productivity shocks on the business cycle. We however find a lot of literature empirically looking at the effect of overall productivity shocks on the informal economy, some of the recent ones include Amin and Okou (2020), Posti and Maiti (2023) and Haanwinckel and Soares (2021). We attribute this gap to the lack of data on informal productivity, especially for a country like India.

In the induced case, the effects of the informal productivity shocks are magnified, because a rise in informal productivity translates to a rise in country risk, which in turn results in a rise in real interest rate. In our model, a rise in the real interest rate expands the informal part of the formal economy. Thus, in the induced case, the combination of the direct and indirect effects of informal productivity shocks leads to its magnified effect. However, it must be noted informal productivity shocks do not affect country risk to the same extent as formal productivity shocks.

We have set up the following flowchart (Figure 17) to sum up the interaction of the shocks and their impact on the formal and informal economy of India. Positive informal productivity shocks and negative formal productivity shocks induce positive country risk shocks. Country risk shocks and foreign real interest rate shocks result in an increase in the real interest rate, which in turn positively impacts the formal sector and negatively impacts the informal sector of India. Formal and informal productivity shocks directly impact the formal and informal economy. Thus, the dependence of country risk shocks on formal and informal productivity shocks amplifies their impact on the formal and informal sectors in India.

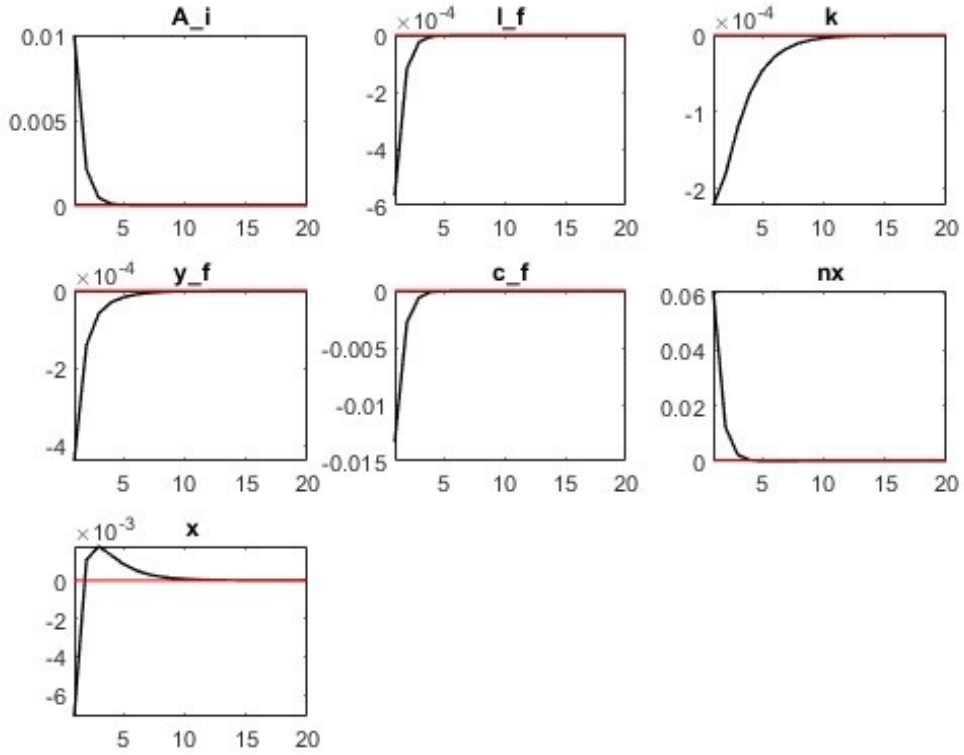

Figure 13: Effect of Informal Productivity Shocks (Independent Case) On Formal Economy

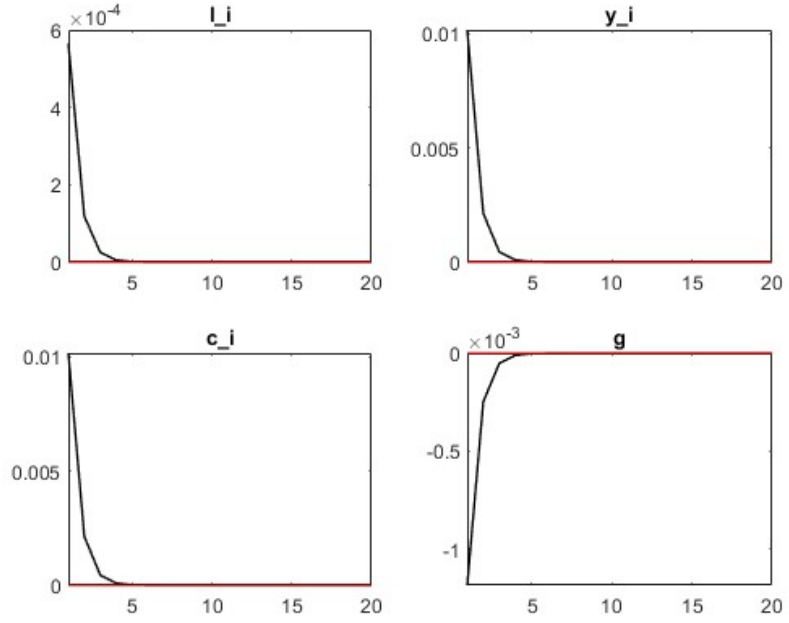

Figure 14: Effect of Informal Productivity Shocks (Independent Case) On Informal Economy and Government Expenditure

A_i refers to informal productivity, R refers to real interest rate, l_f refers to formal labour, k stands for capital,y_f refers to formal output, c_f refers to formal consumption, nx refers to net exports, x refers to investment,l_i refers to informal labour, y_i refers to informal output, c_i refers to informal consumption and g refers to government expenditure.

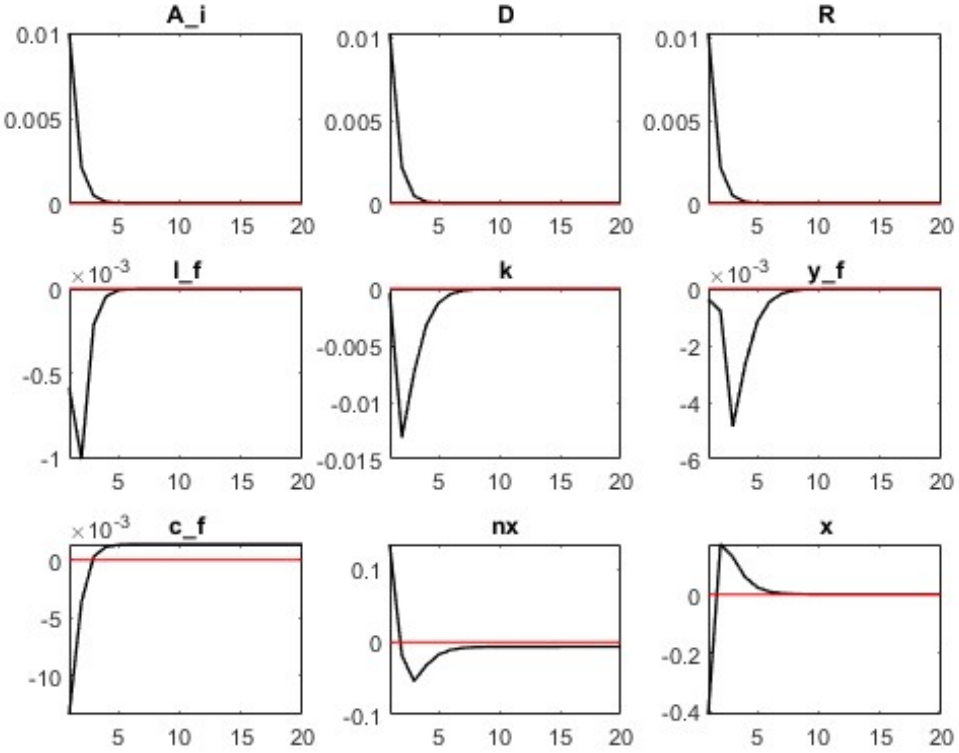

Figure 15: Effect of Informal Productivity Shocks (Informal Productivity Induced Case) On Formal Economy

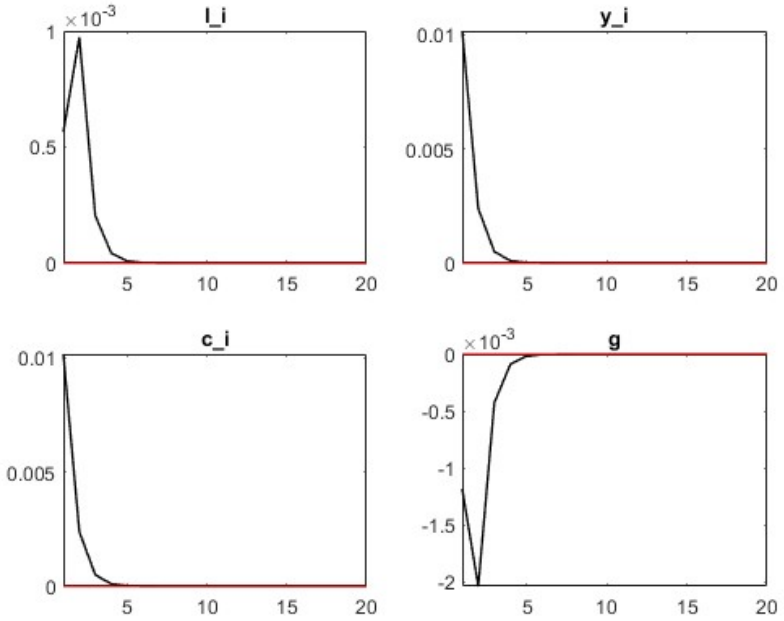

Figure 16: Effect of Informal Productivity Shocks (Informal Productivity Induced Case) On Informal Economy and Government Expenditure

A_i refers to informal productivity, D refers to country risk, R refers to real interest rate, l_f refers to formal labour, k stands for capital,y_f refers to formal output, c_f refers to formal consumption, nx refers to net exports, x refers to investment,l_i refers to informal labour, y_i refers to informal output, c_i refers to informal consumption and g refers to government expenditure.

Figure 17: Transmission of Shocks to the Formal and Informal Economy of India

7 Sensitivity Analysis

7.1 Tax Rate

The income tax rate, one of the most important sources of tax income in India (Dey 2014) varies with different income levels. We chose 10% because it corresponds to the income earned by the majority of Indians involved in formal work. The income tax for the highest income group is 20%. Thus we conduct the analysis again with 20% instead of 10%. When the tax rate is higher, the difference between taxed formal and informal wages is lower. Recalling Equation 25, that means the difference between the supply of formal and informal labourers is not stark. Formal consumption is impacted, with the gap between taxed formal wages and informal wages being lower (refer to Equation 35). Formal capital and formal output are not affected as much as formal labour and formal consumption as the impact is indirect. Labour, government expenditure and formal consumption are the ones that are most affected because of their direct association with taxes, while the other variables are mostly affected indirectly. The signs of all the variables remain the same as the benchmark model of India. Higher tax insulates the fall in government revenue in the face of shocks. This means that raising government taxes can insulate the economy against the impact of shocks. The graphs are shown in section A.4.1 of the appendix.

7.2 Working Capital

We increase the working capital from 0.27 to 1[32] to see how the impact of shocks on the business cycle of India will be affected by the change in parameter. The graphs are shown in section A.4.3 of the appendix.

The higher the working capital parameter, the higher will be the demand for informal labourers. In the face of country risk shocks and foreign interest shocks, it will be costlier to pay back the borrowed amount required for formal labourers. As a result of the one standard deviation of foreign real interest rate shocks or country risk shocks, the magnitude of the fall in formal labour will be higher than the fall in formal labour with lower working capital. Consequently, the rise in informal labour will also be higher but will persist the same. The magnitude of the effect of a fall in foreign real interest rate and country risk is more pronounced for capital too, though not as much as formal labour. Consequently, formal output and government revenue decrease more than the benchmark model. Now that formal labourers are being hired less, formal consumption decreases more than

[32]The reason we choose 1 because, in this model, it represents the maximum possible amount of working capital

it does when working capital is low. The magnitude of the effect on net exports and investment is higher too. Informal output, consumption and labour rise more than they do in the benchmark model.

When country risk does not depend on either informal or formal productivity shocks, there will not be any spillover effect of the two productivity shocks. As a result, the working capital parameter would not affect the impact of productivity shocks on real variables, as changes in working capital can only affect the economy via the real interest rate channel. However, in the induced country risk case, the spillover effects from country risk shocks mean that changes in working capital will impact how both formal and informal productivity affect the business cycle; the effects of formal and productivity shocks on the Indian formal and informal economy become magnified.

7.3 Wage differential

We increase the wage differential parameter from 2.65 to 3[33] to ascertain how the impact of the shocks will be affected if the wage differential increases. The graphs are shown in section A.4.2 of the appendix. Due to foreign real interest rate shocks or country risk shocks, firms now let go of more formal labour because they are more expensive to employ. This results in a transmission of shocks which leads to all the real variables being affected more. However, 3 is slightly higher than 2.65, so the change in the impact of the shocks on the Indian formal and informal economy is not too strong. The higher the wage inequality, the worse the effects of the shocks are.

8 Empirical Methodology

In this section, we focus on the empirical methodology of Mixed-frequency Bayesian VAR. We describe the data. Then we set up the MF-BVAR models with a brief description of the structure of the models and lag length.

8.1 Data Description

In this subsection, we describe the data used for our empirical methodology.

We include data from 1996Q2 to 2016Q4, except for informal output, the formal and informal productivity processes for which we use data from 1996 to 2016.

Though India is characterised by an informal economy, data availability is minimal. We use Dynamic General Equilibrium (DGE) estimates of the proportion of informal output as a percentage

[33]We choose the value 3 because if we recall, we saw from the calibrations that the wage differential for informal sectors as a whole in India was 3, while the 2.65 we have chosen denotes the wage differential for the informal part of the formal economy.

of GDP from 1996 to 2016 as our data for informal output. This data is obtained from Elgin et al. (2021) and is put forward by the World Bank database. Literature previous to Elgin et al. (2021) that have created a database of informal economies include Schneider and Enste (2000), Ihrig and Moe (2004) ,Schneider, Buehn, and Montenegro (2010), C. Elgin, Oztunali, et al. (2012), Ardizzi et al. (2014) and Orsi, Raggi, and Turino (2014). We choose the Dynamic General Equilibrium (DGE) estimates from Elgin et al. (2021) not only because it presents the latest data but also because the DGE approach stands out among estimation methods (N. V. Loayza 2016). Other methods, such as the electricity-demand and currency-demand approaches, suffer from significant limitations[34] (Ahumada, Alvaredo, and Canavese 2007).

We obtain data for government expenditure in India from the Reserve Bank of India[35].

The call money rate[36] is used for nominal interest rates since it is the most appropriate indicator for the real interest rate defined in our DSGE model. We calculate the inflation rate using quarterly data for CPI and we subtract it from the nominal interest rate to obtain the real interest rate[37]. Data for call money rate and Consumption Price Index (CPI) for all items of India is taken from Federal Reserve Economic Data (FRED). The real interest rate is in percentage form. We use the effective funds rate and the US consumer price index from FRED to calculate the foreign real interest rate[38]. Following Neumeyer and Perri (2005) and Equation (3.13), the country risk process is calculated from the difference of the HP filtered values of the Indian and US real interest rate[39]. Deviation of

[34]The electricity-demand model relies on strong assumptions, such as the idea that all informal economic activity requires electricity and that the relationship between informal production and electricity usage remains constant over time—assumptions that are often unrealistic. Similarly, the currency-demand approach assumes that all transactions in the informal sector are conducted in cash and that the base year of analysis has no informal sector activity, both of which are subject to considerable caveats and limitations in accuracy.

[35]Reserve Bank of India has different base prices for different quarters of data, but we use the process splicing such that data are in 1999-2000 constant prices.

[36]Previous literature (Maitra 2018, Mohanty 2012, B. Bhattacharya, Bhanumurthy, and Mallick 2008,Pattanaik and Sahoo 2001) has used this call money rate for India in business cycle analysis. An alternative would be the Marginal Cost of Funds-based Lending Rate (MCLR) which is more used for capital rental price, instead of the definition of real interest rate provided in the DSGE model.

[37]Neumeyer and Perri (2005) states that the subtraction of domestic inflation from nominal interest rate is the ideal method for capturing the real interest rate. However, they could not construct the same for Argentina because of data-related issues, so they used a proxy.

[38]The call money rate in India serves a role similar to the federal funds rate in the United States, as both are key indicators of short-term interest rates that influence monetary policy and liquidity in their respective economies. Both rates are critical for managing working capital and investment costs, as they affect the availability and cost of funds for financial institutions.

[39]For our empirical analysis, we were at first apprehensive about modelling country risk using this method because of the fear the analysis would lead to multicollinearity. However, an intensive literature review made us realise Bayesian Vector Autoregression (VAR) models are considered robust to multicollinearity compared to traditional VAR models because they incorporate prior distributions into the estimation process. These priors act as regularisers by introducing additional information into the model, reducing the variability in parameter estimates that multicollinearity would otherwise amplify. As Tahir (2014) says 'Bayesian methods provide an inherent solution to circumvent the problem

formal and informal productivity from the steady state is calculated using Equations 32 and 33[40], following the recommendation of Tiryaki (2012) in the absence of data availability.

A detailed description of the data has been explained in Appendix A.1.

We have seasonally adjusted our data using STL decomposition. STL (Seasonal-Trend decomposition using Loess) is a technique for breaking down a time series into its constituent parts: seasonal, trend, and residual. The STL method estimates the trend factor, which is a flexible estimate of the underlying structure in the data, using a locally weighted regression technique called Loess.

We use the Hodrick-Prescott (HP) filter to find the cyclical component of our data series by minimising the following equation:

$$\min_{\tau_t}[\sum_{t=1}^{T}(y_t-\tau_t)^2+\lambda\sum_{t=2}^{T-1}((\tau_{t+1}-\tau_t)-(\tau_t-\tau_{t-1}))^2] \tag{44}$$

y_t is the observed value of data at time t, τ_t is the trend component (smoothed series) at time t and λ is the smoothing parameter that controls the trade-off between the trend's smoothness and the cyclical component's variance. When λ is high, the trend is smoother. When λ is low, the filter is more useful for shorter data sizes. We use λ equal to 1600 for quarterly data (Ravn and Uhlig 2002) and 100 for annual data (Backus, Kehoe, and Kydland 1992).

8.2 Mixed Frequency Bayesian Structural Vector Autoregression Model

In this subsection, we explain the mixed-frequency Bayesian VAR (MFBVAR) models used in our analysis

We obtain simulated impulse responses to foreign real interest rate shocks, country risk shocks, and formal and informal productivity shocks for India. We use the signs obtained from the simulated impulse responses to structurally identify the BVAR models and empirically find the impact of the shocks on the business cycle of India, given data from 1996Q2 to 2016Q4. Data for informal output is annual. Since there is a lack of informal data for India, there is a huge gap in the literature regarding business cycle dynamics and informal economy in India, which we aim to close.

We have estimated four BVAR models. One model attempts to see the impact of foreign real interest rate shocks on the real interest rate, government expenditure and informal output of India.

of multicollinearity'. We chose to use this method to best reflect the intuition of the DSGE model in our empirical analysis.

[40]Data for the proportion of formal labour, informal labour, formal output and informal output has been calculated from Elgin et al. (2021). The values of formal and informal exponent have been captured during calibration in Section 5.

The second model attempts to assess the impact of country risk shocks on the real interest rate, government expenditure and informal output of India. The third model attempts to find the impact of formal productivity shocks on country risk, the real interest rate, government expenditure and informal output of India. The fourth model examines the impact of informal productivity shocks on country risk, the real interest rate, government expenditure and the informal output of India.

we look into selecting lags for our mixed-frequency Bayesian VAR (MF-BVAR) models. Enilov (2020) states that the existing MF-VAR literature addresses numerous approaches for selecting lag length, but no parsimonious criterion is proposed. On the one hand, Ghysels, Sinko, and Valkanov (2007), Clements and Galvão (2008), and Motegi and Sadahiro (2018) consider one autoregressive lag to represent potential seasonality. When it comes to mixed frequency VAR, Kuzin, M. Marcellino, and Schumacher (2011), Enilov (2020) and Bai, Ghysels, and Wright (2013) believe that the maximum length should be set at four.

Following the literature, we impose a maximum lag condition of four and use the Akaike Information Criterion (AIC), Hannan-Quinn Information Criterion (HQIC) and Bayesian Information Criterion (BIC) criteria to decide which lag is most suitable for our models.

AIC, first brought forward by Akaike (1974), aims to find the best-fitting model while penalising for additional parameters to avoid over-fitting (Haughton, Oud, and Jansen 1997), with the formula:

$$\text{AIC} = -2 \cdot \ln(L) + 2k \tag{45}$$

where L represents the maximum likelihood and k represents the number of parameters. We choose the lag for which AIC provides the lowest value.

BIC, put forward by Schwarz (1978), uses the formula:

$$\text{BIC} = -2 \cdot \ln(L) + k \cdot \ln(n) \tag{46}$$

where L represents the maximum likelihood function, k represents the parameters and n represents the number of observations. BIC penalises models with additional parameters more aggressively than AIC, especially as the dataset size increases.

HQIC, first used in Hannan and Quinn (1979), uses the formula:

$$\text{HQIC} = -2 \cdot \ln(L) + 2k \cdot \ln(\ln(n)) \tag{47}$$

where Lrepresents the maximum likelihood function, k represents the parameters, and n represents the number of observations. HQIC penalises model complexity less than BIC but more than AIC.

Lower values of AIC, BIC, and HQIC indicate better models. We have four MF-BVAR models.

For all the four models, the optimal lag length came to be four using all three critera.

Data on informality in India is limited. We have a mixture of high-frequency and low-frequency data. Data on informal output, formal productivity and informal productivity are annual. Government spending, foreign real interest rate and country risk are in quarterly terms.Given the mismatch of data frequency Bayesian VAR is the best option for us to pursue so that we do not face a proliferation of parameters[41].

Bayesian VAR (BVAR) models have the same mathematical form as a VAR model. The difference in the two methodologies lies in the way the parameters are estimated. In a standard VAR model, the parameters are estimated using the OLS approach. Following Koop, Korobilis, et al. (2010), researchers worry about how to build models that are flexible empirically but do not have over-parameterisation problems. Koop, Korobilis, et al. (2010) argue shrinkage can be a solution to this problem and a way of introducing that would be to impose restrictions on parameters, which has led to the large use of Bayesian methods. Bayesian techniques do not have the problem with the loss of degrees associated with the classical VAR model and hence can be used to provide parameter estimates where there is comparatively little data. Support in favour of Bayesian VAR can be found in Dijk (2011) and O'Hagan and West (2010). Furthermore, Bayesian VAR helps with the problems of multicollinearity, following Tahir (2014).

We perform Bayesian VAR analysis with a multivariate Normal-Inverse Wishart conjugate prior, which indicates that the joint prior and posterior distributions of parameters in the sampling distribution exist in the same parametric distribution family. The benefit of using a conjugate prior distribution is not only the fact that the most difficult part of the Bayesian process, integration, can be avoided but also that because the prior and posterior distribution reside in the same parametric distribution family such that the inferential process is just a case of updating the parameters in the prior distribution, reducing computing costs significantly (Chang 2002). Furthermore, Tahir (2014) uses log predictive likelihood to analyse Minnesota Priors, Independent Normal–Wishart Priors and Independent Minnesota–Wishart Priors in Bayesian VARs and finds that Independent Normal–Wishart Priors tend to have the most robust results.

We are considering the multivariate Normal-Inverse Wishart (MN-IW) prior. We first consider the autoregressive parameters of the reduced VAR model. In model 1 (Equation 36), the autoregressive parameters are denoted by A. In this particular Bayesian framework, autoregressive

[41]Following the literature on mixed frequency, two methods are normally used for mixed frequency VAR estimation - UMIDAS and Bayesian VAR. UMIDAS does not perform well when there is less data. Our dataset is relatively small while we have high number of parameters. We have 4 lags for each model. For example, for model 1 we have 5 variables and a constant. The degree of freedom is 105. Thus Bayesian VAR is the best option.

parameters $A \sim N(A_0, \Sigma \otimes V)$ where the distribution function is given by

$$(2\pi)^{\frac{-nk}{2}} |\Sigma|^{\frac{-k}{2}} |V|^{\frac{-n}{2}} exp\{-\frac{1}{2} tr[(A - A_0)V^{-1}(A - A_O)^{-1}]\} \tag{48}$$

where A_0 is the mean of A, Σ is the residual covariance matrix of the VAR, which describes the covariance structure between the errors of different variables in the system, V is a covariance matrix that is specific to the autoregressive parameters, n is the number of equations (variables) in the VAR system, k is the number of lagged variables, tr denotes the trace operator, $|\Sigma|$ is the determinant of the covariance matrix Σ and $|V|$ is the determinant of the matrix V.

Σ follows Inverse Wishart distribution IW (Σ_0, d) given by:

$$IW(\Sigma_0, d) = \frac{|\Sigma_0|^{\frac{-d}{2}}}{2^{\frac{-d}{2}} \Gamma_n(\frac{d}{2})} |\Sigma|^{\frac{-(d+n+1)}{2}} exp\{-\frac{1}{2} tr[\Sigma_0 \Sigma^{-} 1]\} \tag{49}$$

where Σ_0 is the scale matrix, d controls the strength of the prior and Γ_n is the multivariate gamma function.

Following previous literature, we assume the prior for the auto-regressive parameters A_0 is centred at zero with a diagonal covariance matrix V and the prior for the residual covariance matrix is inverse Wishart with a unitary diagonal matrix as scale and n+1 degrees of freedom in a conjugate setting. This setup assumes equal variances and no prior correlations between variables, enabling flexible inference driven by the data. A Gibbs sampler algorithm (Gelfand and Smith 1990) has been used to obtain draws from the posterior distributions.

We lead a discussion on the 'mixed-frequency' component of our analysis. The state space-based mixed-frequency Bayesian VAR was first proposed by Schorfheide and Song (2015) using a Minnesota-style normal inverse Wishart prior. In Ankargren and Jonéus (2019), another mixed-frequency Bayesian VAR was considered but with a steady-state prior. 'State space' representation was first proposed by Kalman who presented an algorithm with a set of rules to sequentially forecast and update a set of projections of the unknown state vector. There are two common aggregations when it comes to mixed frequency VAR, that have been used in the literature: intra-quarterly averaging and triangular aggregation. We use the intra-quarterly averaging by Schorfheide and Song (2015).

We use four BVAR models for our analysis. To explain the set-up of the models, we currently focus on Model 1; the BVAR(4) model with four variables: foreign real interest rate, Indian real interest rate, government expenditure and informal output of India.

Equation 44 gives us the structural equation from which we derive the reduced form equation in Equation 45.

$$B_{20}Z_t = c^*_{20} + B_{21}Z_{t-1} + B_{22}Z_{t-2} + B_{23}Z_{t-3} + B_{24}Z_{t-4} + \nu_t \tag{50}$$

where $Z_t = (FI_t, INT_t, GOVT_t, INF_t)'$, where $FI_t, INT_t, GOVT_t, INF_t$ are the cyclical decomposition of HP filtered values for real foreign interest rate, Indian real interest rate, government expenditure and informal output of India.

The errors ν_t are structural disturbances and are assumed to be serially uncorrelated. ν and c^*_{20} are 4x1 vectors while B_{20} is a 5x5 matrix.

If we pre-multiply the model with B^{-1}_{20}, we get

$$Z_t = c_{20} + A_{21}Z_{t-1} + A_{22}Z_{t-2} + A_{23}Z_{t-3} + A_{24}Z_{t-4} + e_t \tag{51}$$

where $c_{20} = B^{-1}_{20}c_{20}{}^*$, $A_{2s} = B^{-1}_{20}B_{2s}$ where s runs from 1 to 4 , and $e_t = B^{-1}_{20}\nu_t$

This is identical to reduced standard VAR, which is required to calculate the impulse responses of shocks related entirely to, say, the foreign real interest rate. We identify the model with the help of sign restrictions obtained from the simulated impulse response. For this particular MF-BVAR model, our DSGE model has indicated a rise in foreign real interest rate leads to a rise in Indian real interest rate and informal output compared to total GDP and a decline in government spending.

The second BVAR(4) model has four variables: Country risk of India, Indian real interest rate, government spending of India and informal output of India. The third BVAR(4) model has the following variables: formal productivity of India, country risk of India, Indian real interest rate, government spending of India and informal output of India. The fourth BVAR(4) with five variables: informal productivity of India, country risk of India, real interest rate of India, government spending and informal output of India. The VAR equations of these three models have been set up in Appendix A.3.

We set up the sign restrictions in Table 3. Signs restrictions are based on the signs of the simulated impulse responses from the DSGE Model. When country risk and foreign interest rate increase, real interest rate rises, following Equation 21. When the real interest rate rises, firms hire more informal labour compared to formal labour, given the contract with formal labour. The contract states that the firms have to pay a percentage of wages to formal labourers before production begins; firms have to take out a working capital loan to fulfil the contract. When the real interest rate rises, firms have to pay back a higher amount of loan, thus firms prefer to employ informal labour. This leads to an increase in informal output. Tax collection falls as formal labour decreases. Given the balanced budget condition, government spending falls. When there are positive informal productivity shocks, informal labour and, hence, informal output rises. Total tax collection decreases, government revenue decreases, and government expenditure

decreases, following a balanced budget. In the induced country risk model, the increase in informal productivity leads to a rise in country risk, which further increases the real interest rate and amplifies the impact of the informal productivity shocks. Similarly, in the presence of positive formal productivity shocks, formal labour increases, government revenue increases and hence government expenditure increases; informal labour falls and hence informal output falls. However, in the induced country risk model case, formal productivity shocks lead to a fall in country risk, which further leads to a fall in the real interest rate and amplifies the effects of the formal productivity shocks.

Shocks / Variables	Foreign Real Interest Rate	Country Risk	Real Interest Rate	Formal Productivity	Informal Productivity
Country Risk	N/A	N/A	N/A	-	+
Real Interest Rate	+	+	N/A	-	+
Government	-	-	-	+	-
Informal Output	+	+	+	-	+

Table 3: Sign Restrictions

We perform Bayesian VAR analysis with multivariate normal-inverse Wishart conjugate prior. The prior and posterior distributions exist in the same parametric distribution family for conjugate priors and the inferential process just involves updating the parameters in the prior distribution, reducing computing costs (Chang 2002). Furthermore, Tahir (2014) finds this process to be the most robust among other Bayesian priors for Bayesian VAR analysis. We follow the same setup of the prior as in Section 2.38.

9 Impulse Response Functions

In this section, we present the impulse responses obtained from the Mixed-frequency Bayesian VAR analysis with sign identifications.

9.1 Effect of Foreign Real Interest Rate Shocks

In this subsection, we look into the impact of transmission of foreign real interest rate shocks on the Indian informal economy. First, we consider Model 1, a BVAR (4) model, with foreign real interest rate, Indian real interest rate, government spending and informal output of India. We are identifying the model with the help of the sign restrictions obtained from the DSGE model. We refer to Figure 18 for this particular analysis.

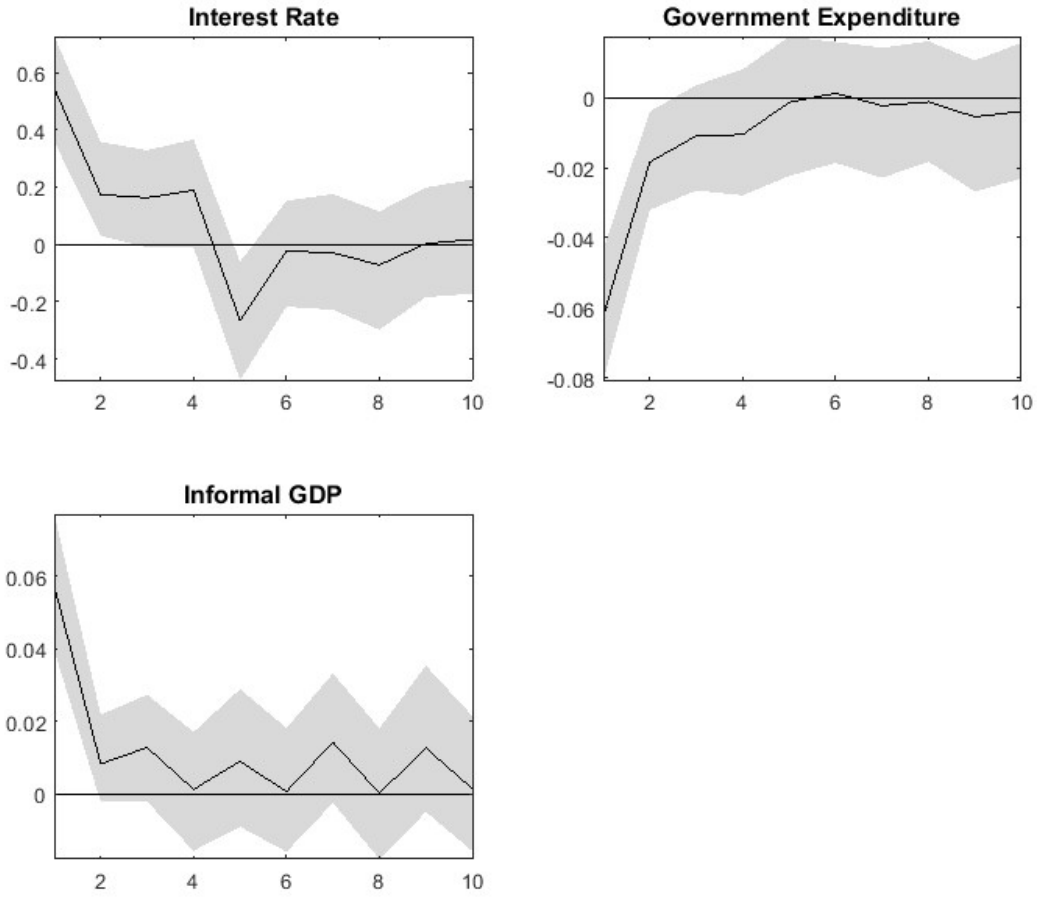

Figure 18: Effect of Foreign Real Interest Rate Shocks: Empirical Analysis

Given data from 1996Q2 to 2016Q4, a rise in the foreign real interest rate leads to an increase in the Indian real interest rate and informal GDP. It leads to a decrease in government expenditure in India. Following the DSGE model, when the foreign real interest rate rises, the real interest rate increases, which makes it harder to pay the working capital loan required for formal employment. As such, informal labour rises and formal labour falls, leading to a fall in government expenditure and a rise in informal output. On impact, real interest rate increases by 0.6 while government expenditure decreases by 0.06 and informal output increases by 0.06. This is in line with the simulated impulse responses, because in the DSGE model, we assume the foreign real interest rate shocks have a direct effect on the real interest rate. Foreign real interest rate shocks affect government expenditure and informal output through the real interest rate channel. The effect of

the shocks on all the variables seems to be credible[42] for around 2 periods.

The informal sector has been relatively ignored in the literature. There has been research focussing on the impact of nominal interest rate shocks on informal sectors in emerging countries. We discuss them to compare how real interest rate shocks impact the informal sector compared to nominal interest rate shocks. Using panel local projection models for 19 emerging countries from 1997Q4 to 2019Q4, Arteta, Kamin, and Ruch (2022) show that rising US nominal interest rates cause emerging countries to cut government budgets, which is also shown in our results and can cause a financial crisis. Iacoviello and Navarro (2019) conclude from his dataset of 25 emerging and 25 developed countries from 1965Q1 to 2016Q2 that spillover of US nominal interest rate increases the vulnerability of the emerging countries, unlike developed countries.

9.2 Effect of Country Risk Shocks

In this subsection, we look into the impact of the transmission of country risk shocks on the Indian informal economy. We consider Model 2, a BVAR (4) model, with the country risk of India, Indian real interest rate, government spending and informal output of India. We are identifying the model with the help of the sign restrictions obtained from the DSGE model. We refer to Figure 19 for this impulse response analysis.

The impulse response functions show country risk shocks lead to an increase in the Indian real interest rate and informal GDP in India and a decrease in government expenditure. Following the DSGE model, an increase in country risk increases the real interest rate which, similar to the effects of foreign real interest rate shocks, makes it harder to pay the working capital loan required for formal employment. As a result, there is an increase in informal labour and a decrease in formal labour. A decrease in formal labour translates to less government revenue as formal labour pays taxes. Given a balanced budget condition, a fall in government revenue leads to a decrease in government expenditure. The increase in informal labour leads to a rise in informal production.

On impact, real interest rate increases by 0.6 while government expenditure decreases by 0.06 and informal output increases by 0.06. The effect of shocks on all the variables seems to be credible around 2 periods. The results from country risk shocks (Figure 19) are very similar to results from foreign real interest rate shocks (Figure 18).

We discuss some relevant literature. According to Xu, Lv, and Xie (2018) who use annualised panel data for 131 countries spanning 1999–2007, country risk is a robust and significant predictor of the informal economy, with a 1% increase in the country risk rating (a decrease in the country risk) translating into a 0.1% decline in the informal sector. N. Loayza et al. (2007) use cross-country

[42]By credible, we mean that there is a 68% probability that the impact of the shock is negative/positive and is not converging to zero.

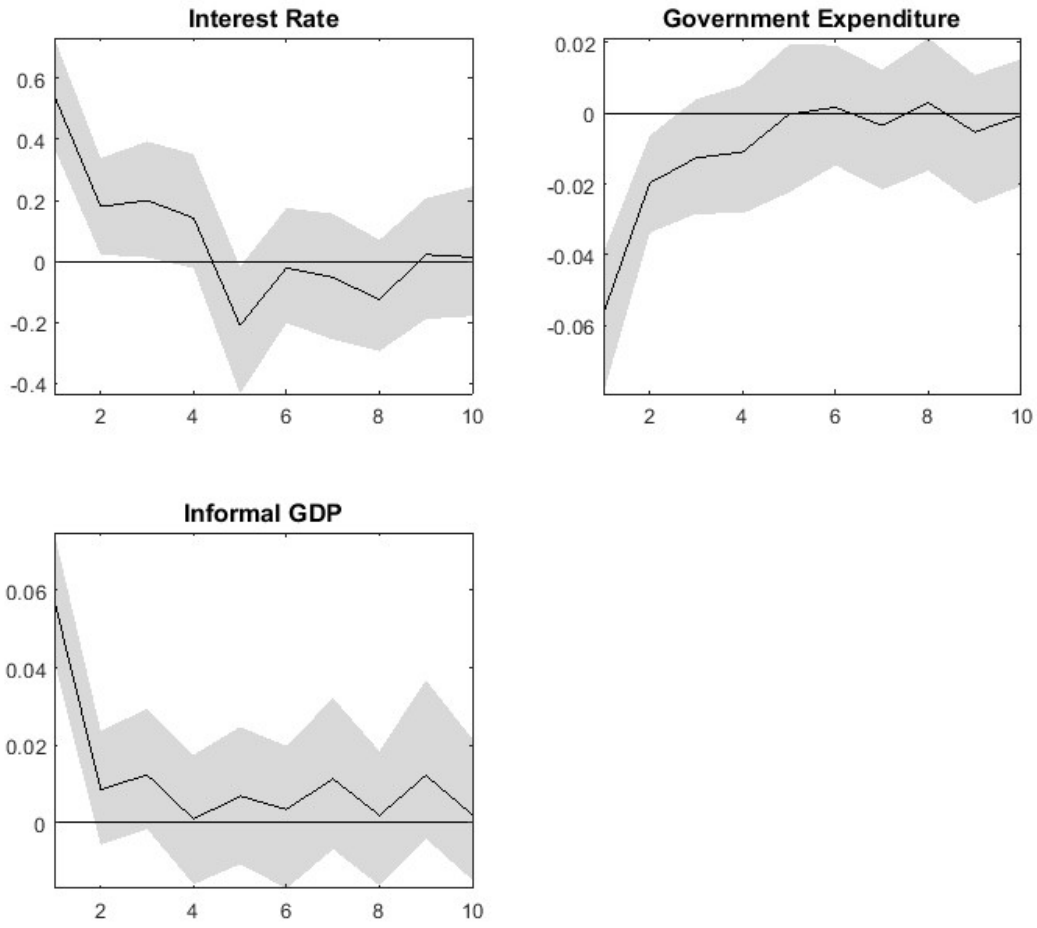

Figure 19: Effect of Country Risk Shocks: Empirical Analysis

regression analysis from a period of 1985 to 2004 in Peru to find informality is significantly negative to country risk, where the country risk is measured with law and order and regulatory freedom. While some previous research has primarily focused on the implications of how country risk affects informality, our study explores the relationship in a business cycle context. By doing so, we contribute to the macroeconomic understanding of the interplay between country risk and the informal economy.

9.3 Effect of Formal Productivity Shocks

In this subsection, we look into the impact of the transmission of formal productivity shocks on the Indian economy. We consider Model 3, a BVAR(4) model, with the following variables: formal

productivity level of India, country risk of India, Indian real interest rate, government spending of India and informal output of India. We are identifying the model with the help of the sign restrictions obtained from the DSGE model. We refer to Figure 20 for this impulse response analysis.

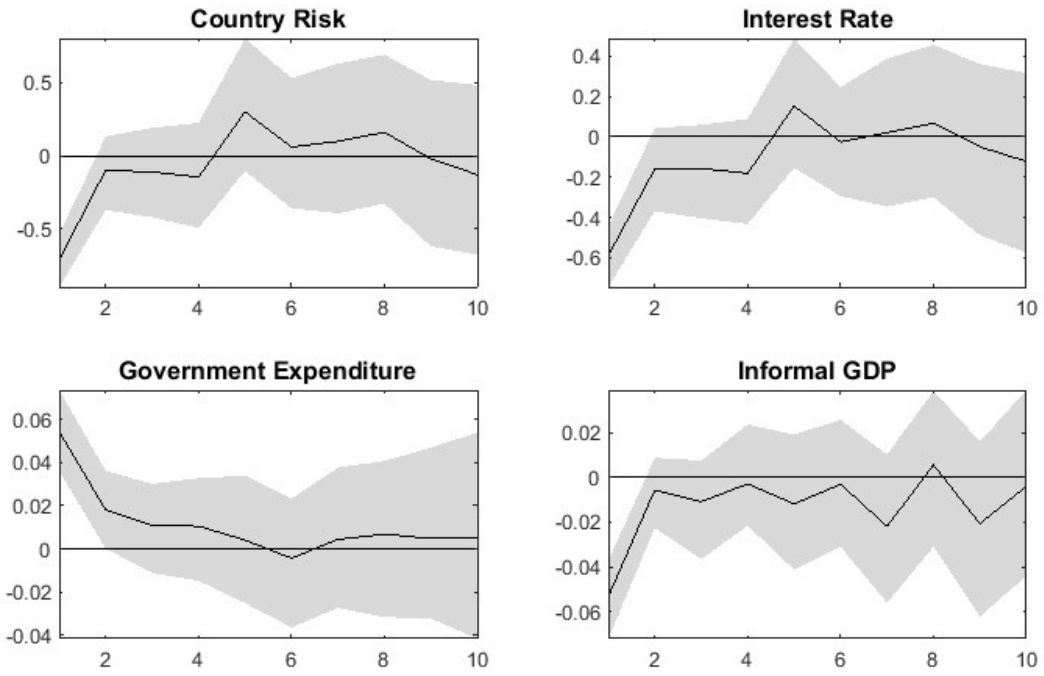

Figure 20: Effect of Formal Productivity Shocks: Empirical Analysis

Formal productivity shocks negatively affect both country risk and the real interest rate. On impact, the country risk decreases by 0.06 to 0.08 while the real interest rate decreases by 0.06. This follows the transmission mechanism in our model.

Our model states that formal productivity shocks result in a decrease in country risk, which leads to a fall in the real interest rate. This fall in real interest rate leads to lower working capital loans, which, given the contract with formal labour[43], results in the hiring of more formal labour. The rise in government revenue arising from more formal labourers being hired[44]leads to a rise in government spending and a fall in informal output. Thus, the impact of formal productivity shocks is amplified by their association with country risk shocks.

[43]Recall, forms have a contract with formal labour that firms need to pay a portion of their wages before production. In order to do that, firms need to obtain a working capital loan.

[44]Formal labourers pay tax. Informal labourers do not.

On impact, government expenditure increases by 0.05 and informal output falls by 0.05. The effects are credible for around 2 periods.

We discuss some literature that might be relevant to our analysis. Chong, Galdo, and Saavedra (2008) conducts a probit analysis with informal employment data of Peru from 1986 to 2001 to conclude that there exists a negative relationship between productivity level and the informal sector in Peru. Opondo, Etyang, and Ayieko (2022) conducts ordinary least square regressions using data of Kenya from 1976 to 2016 to check if the presence of the informal sector affects total factor productivity in Kenya and finds a negative relationship between the two. C. Elgin and Birinci (2016) analyse 161 countries from 1950 to 2010 using panel fixed-effects regression to conclude that in developing countries total factor productivity and informality have a negative relationship, unlike in developed countries.

9.4 Effect of Informal Productivity Shocks

In this subsection, we look into the impact of the transmission of informal productivity shocks on the Indian economy. We consider Model 4, a BVAR(4) model, with variables informal productivity level of India, country risk of India, Indian real interest rate, government spending of India and informal output of India. We are identifying the model with the help of the sign restrictions obtained from the DSGE model. We refer to Figure 21 for this impulse response analysis.

Informal productivity shocks lead to a positive effect on country risk and the real interest rate. On impact, country risk increases by 0.7, real interest rate increases by 0.6, government expenditure falls by 0.05 and informal output rises by 0.05. The effects are credible for around 2 periods.

The magnitude of the impact of the shocks can be explained by the transmission mechanism explained in the DSGE model. Informal productivity shocks raise country risk foremost, which further leads to a rise in the real interest rate. Real interest rate shocks increase working capital loans, which, in the presence of the contract, leads to the hiring of more informal labour. Less hiring of formal labour leads to decreased tax revenue. Given the balanced budget condition, government revenue falls. There is a rise in informal employment, accompanied by a rise in informal production. The impact of informal productivity shocks on the Indian economy is amplified.

Our results are similar to that of Fiess, Fugazza, and Maloney (2010) which conduct a DSGE analysis with five Latin American countries from the time period of 2005 to 2019 to state informal productivity shocks are essential to account for changes in the informal market (self-employment portion of the informal sector) in the face of a pandemic.

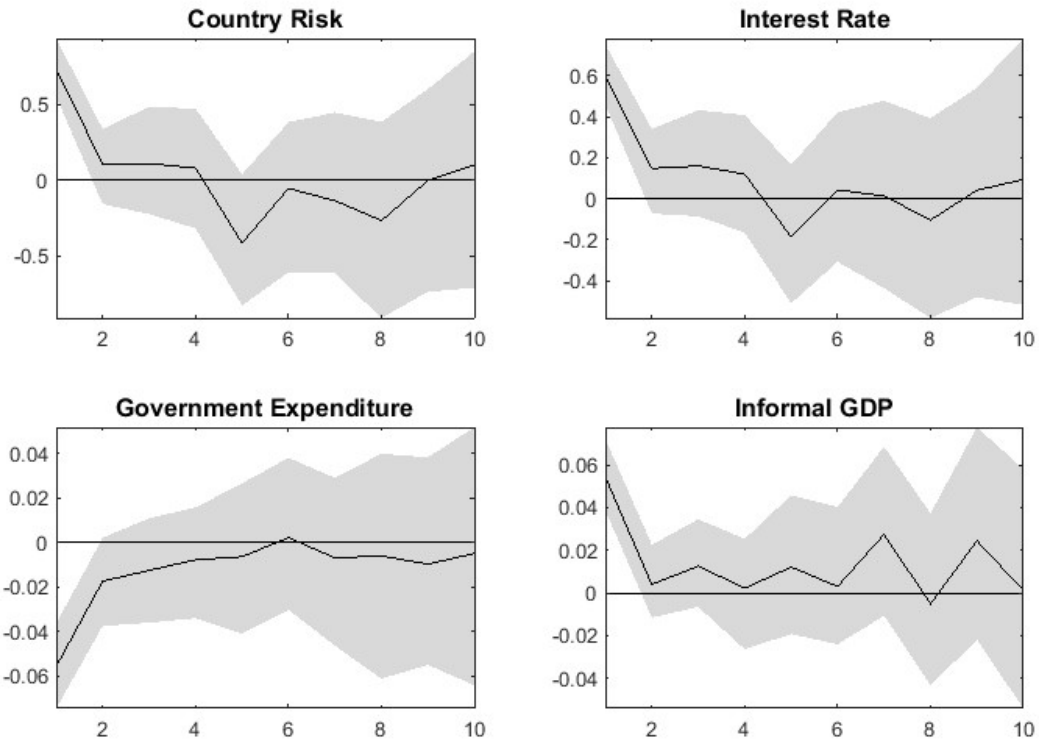

Figure 21: Effect of Informal Productivity Shocks: Empirical Analysis

9.5 Robustness Checks

We conduct a robustness check for all the four BVAR models considered by changing the lag length for all the VAR models to 2 and There have been little to no changes in the impulse responses. The graphs are shown in Sections A.5.1 and A.5.2 of the appendix.

10 Summary and Conclusions

To summarise, we extend the initial model of Neumeyer and Perri (2005) to include the concept of the informal sector in the model. Informality can be defined in various ways. Recently there has been a surge in informalisation of the formal sector in India, where formal sectors have started employing informal labour in India (Hammer 2019, Abraham 2017, Kapoor and Krishnapriya 2023, Singhari and Madheswaran 2017). The lack of literature on this type of informal sector in business cycle analysis has motivated us to consider it for our model.

Our model consists of firms, households and a government sector. Firms depend on a working capital loan to pay a portion of their wages to formal sectors before production begins, based on

a contract. However, no such contract exists for informal labour as they are mostly devoid of any employment-related benefits. Households supply both formal and informal labour. Formal labourers pay income taxes, informal labourers do not. Government revenue is decided by income tax. Given a balanced budget condition, government revenue is equivalent to government expenditure.

The real interest rate associated with the working capital loan is decomposed into a foreign real interest rate and country risk component. We explore three scenarios where country risk depends on independent causes, country risk can be induced by formal productivity shocks and country risk can be induced by informal productivity shocks.

We add to the original model of Neumeyer and Perri (2005) in the following ways: (1) inclusion of formal and informal labour with different wage rates; (2) inclusion of a contract with formal labour which states firms need to pay formal labour a portion of wages before production, unlike informal labour; (3) modelling of country risk depending on formal and informal productivity separately; (4) inclusion of taxation and the government sector.

We find that positive foreign real interest rate shocks and country risk shocks result in a fall in formal employment and a rise in informal employment. Formal capital, investment, formal output, and government revenue fall. Formal consumption falls at first but increases in later periods following the Euler Equation. Informal output and informal consumption rise. This suggests that positive real interest rate shocks result in the contraction of the formal sector and the expansion of the informal sector.

Foreign real interest rate shocks are more persistent but less volatile than country risk shocks. The signs of impulse responses are the same irrespective of whether the country risk shocks are due to independent causes, formal productivity shocks or informal productivity shocks. Country risk shocks are the least persistent when they originate from independent causes. Country risk shocks are more dependent on formal productivity than informal productivity.

For India, formal productivity always leads to an expansion of the formal sector and a contraction of the informal sector. Informal productivity shocks lead to a contraction in the formal sector and an expansion of the informal sector. When formal productivity rises, country risk decreases for India. When informal productivity rises, country risk increases for India. Country risk responds to changes in formal productivity more than changes in informal productivity. The dependence of country risk on formal and informal productivity shocks leads to the amplified impact of shocks on the formal and informal economy of India. In India, informal productivity is less volatile than formal productivity, which previous literature (Leyva and Urrutia 2020, Coşkun 2022) have found for Mexico.

We estimate a Mixed Frequency Bayesian VAR identified using sign restrictions obtained from the DSGE model. We estimate four BVAR models. One model attempts to see the impact of foreign

real interest rate shocks on the real interest rate, government expenditure and informal output of India. The second model attempts to assess the impact of country risk shocks on the real interest rate, government expenditure and informal output of India. The third model attempts to find the impact of formal productivity shocks on country risk, the real interest rate, government expenditure and informal output of India. The fourth model examines the impact of informal productivity shocks on country risk, the real interest rate, government expenditure and the informal output of India. We find that the optimal lag for all four models is four. We conduct robustness checks with lags two and three. We assume a multivariate Normal-Inverse Wishart prior distribution.

We find that foreign real interest rates and country risk shocks lower government expenditure and raise the real interest rate of India and the proportion of informal output. Country risk and, subsequently, the real interest rate are negatively affected by formal productivity and positively affected by informal productivity. Country risk and foreign real interest rate shocks impact the real interest rate the most, followed by other variables, in line with findings from the DSGE model.

Our main conclusion is that the presence of informal sectors helps curb the impact of most shocks on the business cycle of India, acting as a safety net, which goes with the intuition of Horvath (2018) and Marjit, Bhattacharyya, and L. Yang (2023). Policymakers often view informality as a detriment to the development of emerging countries, focusing on the lack of regulation, tax evasion, and job insecurity associated with informal labour. However, the ability of the informal sector to absorb shocks highlights its important role in India's economy. Policymakers need to adopt a more nuanced approach that incorporates both the formal and informal sectors into their strategies, making them more inclusive.

However, it must be remembered that informal productivity shocks do induce country risk shocks in India. This highlights a critical implication of our research: real interest rate fluctuations may stem from disturbances linked to the informal sector. Therefore, effective regulation of the informal sector is important. This brings us back to our original policy implication, ie. policymakers need to consider the role of informal sectors instead of overlooking them.

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A.1 Detailed Data Description

- **India Nominal Interest Rate:** Source: Federal Reserve Economic Data.
  Code: IRSTCI01INQ156N.
  Description: Immediate Rates: Less than 24 Hours: Call Money/Interbank Rate for India, Percent, Quarterly, Not Seasonally Adjusted.

- **India Consumer Price Index:** Source: Federal Reserve Economic Data.
  Code: INDCPIALLQINMEI.
  Description: Consumer Price Index: All Items for India, Quarterly, Not Seasonally Adjusted.
  Note: We calculate the inflation rate (growth rate same period previous year) from this data. We subtract the inflation rate from the nominal interest rate to obtain the real interest rate for India.

- **US Nominal Interest Rate:** Source: Federal Reserve Economic Data.
  Code: FEDFUNDS.
  Description: Effective Federal Funds Rate, Percent, Monthly, Not Seasonally Adjusted. Note: We manually adjusted the data quarterly.

- **US Consumer Price Inflation Rate:** Source: Federal Reserve Economic Data.
  Code: CPALTT01USQ659N.
  Description: Consumer Price Index: Total All Items for the United States, Growth Rate Same Period Previous Year, Quarterly, Not Seasonally Adjusted.
  Note: Since the CPI data is already 'Growth Rate Same Period Previous Year', we subtract the US inflation rate directly from US nominal interest rate to get US real interest rate.

- **Government Expenditure:** Source: Handbook of Statistics on Indian Economy, 2020-21, Reserve Bank of India. Table No. 161.
  Title: Quarterly Estimates Of Gross Domestic Product (At Constant Prices).
  Note: We use the process splicing to fix the model at constant prices at 1999-2000.

- **Informal Output:** Source: World Bank Database, extracted from Elgin et al. (2021)
  Title: Dynamic general equilibrium model-based (DGE) estimates of informal output (% of official GDP), Annual.

A.2 Model

Consider the following model:

A.2.1 Model: Household side

$$u(c_f, c_i, l_f, l_i) = \frac{1}{1-\sigma}[c_{f_t} + c_{i_t} - \psi_1 l_{i_t}^{v_1} - \psi_2 l_{f_t}^{v_2}]^{1-\sigma} \tag{A.52}$$

Budget constraint:

$$c_{f_t} + c_{i_t} + x_t + b_t + \kappa(b_t) \leq l_{i_t} w_{i_t} + (1-\tau) l_{f_t} w_{f_t} + R_{t-1} b_{t-1} - r_t k_{t-1} \tag{A.53}$$

where,
c_{f_t}= formal consumption
c_{i_t}= informal consumption
x_t=investment
c_{f_t}= relative price of informal consumption
b_t=bonds
$\kappa(b_t)$ is a convex function and denotes the cost of holding bonds.
l_{f_t}=formal labour
l_{i_t}=informal labour
w_{f_t}= formal wage
w_{i_t}= informal wage
r_t=capital interest rate
k_{t-1}=capital
R_{t-1}=bonds interest rate
τ= tax rate

Bond Holding cost:

$$\kappa(b_t) = \frac{\kappa}{2} y_{f_t} [\frac{b_t}{y_{f_t}} - \bar{b}]^2 \tag{A.54}$$

κ is a constant which denotes the size of the bond holding costs and $\bar{b}$ is the steady state level of the ratio of bonds to formal GDP.
Investment:

$$x_t = k_t - (1-\delta) k_{t-1} + \Phi(k_{t-1}, k_t) \tag{A.55}$$

where $\Phi(k_{t-1}, k_t)$ is the adjustment cost which has been used to avoid excessive volatility of

investment and δ is the depreciation rate.
Adjustment cost Function:

$$\Phi(k_{t-1}, k_t) = \frac{\phi}{2} k_{t-1} [\frac{(k_t - k_{t-1})}{k_{t-1}}]^2 \tag{A.56}$$

where ϕ is the capital adjustment cost parameter.
We are considering the proportion of labour such that

$$l_{i_t} + l_{f_t} = c \tag{A.57}$$

$$\hat{l}_{f_t} = -\hat{l}_{i_t} \tag{A.58}$$

We assume

$$\frac{w_{f_t}}{w_{i_t}} = \mu \tag{A.59}$$

A.2.2 Lagrangian

$$L = \frac{1}{1-\sigma}[c_{f_t} + c_{i_t} - \psi_1 l_{i_t}^{v_1} - \psi_2 l_{f_t}^{v_2}]^{1-\sigma} + \lambda_{f_t}[l_{i_t} w_{i_t} + (1-\tau) l_{f_t} w_{f_t} + r_t k_{t-1} + R_{t-1} b_{t-1} - c_{f_t} - c_{i_t} - x_t - b_t - \kappa(b_t)] \tag{A.60}$$

A.2.3 Labour: Household side

Using A.9,

$$L = \frac{1}{1-\sigma}[c_{f_t} + c_{i_t} - \psi_1 l_{i_t}^{v_1} - \psi_2 l_{f_t}^{v_2}]^{1-\sigma}$$
$$+\lambda_{f_t}[l_{i_t} w_{i_t} + (1-\tau) l_{f_t} w_{f_t} + r_t k_{t-1} + R_{t-1} b_{t-1} - c_{f_t} - c_{i_t} - x_t - b_t - \kappa(b_t)]$$

Using A.6 and A.8,

$$\frac{\delta L}{\delta l_{f_t}} = -\psi_1 v_1 (1 - l_{f_t})^{v_1 - 1} + \psi_2 v_2 l_{f_t}^{v_2 - 1} - (1 - \tau - \frac{1}{\mu_i}) w_f \tag{A.61}$$

At optimum, A.10 is equal to zero.

FOC:

$$-\psi_1 v_1 (1 - l_{f_t})^{v_1 - 1} + \psi_2 v_2 l_{f_t}^{v_2 - 1} = (1 - \tau - \frac{1}{\mu_i}) w_f \tag{A.62}$$

A.2.4 Capital: Household side

From equation A.4, A.5 and A.9, we get,

$$L = \frac{1}{1-\sigma}[c_{f_t} + c_{i_t} - \psi_1 l_{i_t}^{v_1} - \psi_2 l_{f_t}^{v_2}]^{1-\sigma} + \lambda_{f_t}[l_{i_t} w_{i_t} + (1-\tau) l_{f_t} w_{f_t} + r_t k_{t-1} + R_{t-1} b_{t-1} - c_{f_t} - c_{i_t}$$

$$-k_t + (1-\delta)k_{t-1} - \frac{\phi}{2} k_{t-1}[\frac{(k_t - k_{t-1})}{k_{t-1}}]^2 - b_t - \kappa(b_t)] \tag{A.63}$$

$$\frac{\delta L}{\delta k_{t-1}} = \lambda_{f_t} r_t - \lambda_{t-1} - (1-\delta)\lambda_t + \lambda_{f_t} \frac{1}{2(k_{t-1}^2)}(k_t - k_{t-1})^2 +$$

$$\lambda_{f_t} \frac{1}{(k_{t-1})}(k_t - k_{t-1}) - \lambda_{t-1} \frac{1}{(k_{t-2})}(k_{t-1} - k_{t-2}) \tag{A.64}$$

$$\frac{\lambda_{t-1}}{\lambda_t} = \frac{r_t - 1 + \delta + \frac{1}{(k_{t-1})}(k_t - k_{t-1}) + \frac{1}{2(k_{t-1}^2)}(k_t - k_{t-1})^2}{1 + \frac{1}{(k_{t-2})}(k_{t-1} - k_{t-2})} \tag{A.65}$$

From equation A.17, we get,

$$\frac{R_{t-1}}{[1 + \kappa[\frac{b_{t-1}}{y_{t-1}} - \bar{b}]]} = \frac{r_t - 1 + \delta + \frac{1}{(k_{t-1})}(k_t - k_{t-1}) + \frac{1}{2(k_{t-1}^2)}(k_t - k_{t-1})^2}{1 + \frac{1}{(k_{t-2})}(k_{t-1} - k_{t-2})} \tag{A.66}$$

Following Neumeyer and Perri (2005) and Tiryaki (2012), we will disregard the impact of bonds given κ tends to be negligible.

A.2.5 Formal Consumption: Equilibrium

Considering, A.3 and A.9,

$$L = \frac{1}{1-\sigma}[c_{f_t} + c_{i_t} - \psi_1 l_{i_t}^{v_1} - \psi_2 l_{f_t}^{v_2}]^{1-\sigma}$$

$$+\lambda_{f_t}[l_{i_t} w_{i_t} + (1-\tau) l_{f_t} w_{f_t} + r_t k_{t-1} + R_{t-1} b_{t-1} - c_{f_t} - c_{i_t} - x_t - b_t - \frac{\kappa}{2} y_{f_t}[\frac{b_t}{y_{f_t}} - \bar{b}]^2] \tag{A.67}$$

With respect to formal consumption,

$$\frac{\delta L}{\delta c_{f_t}} = [c_{f_t} + c_{i_t} - \psi_1 l_{i_t}^{v_1} - \psi_2 l_{f_t}^{v_2}]^{-\sigma} - \lambda_{f_t} \tag{A.68}$$

At optimum it is equal to zero,

$$[c_{f_t} + c_{i_t} - \psi_1 l_{i_t}^{v_1} - \psi_2 l_{f_t}^{v_2}]^{-\sigma} = \lambda_{f_t} \tag{A.69}$$

With respect to bonds,

$$\frac{\delta L}{\delta b_t} = \lambda_{f_{t+1}} R_t - \lambda_{f_t} - \lambda_{f_t} \frac{\kappa}{2} y_t 2[\frac{b_t}{y_t} - \bar{b}] \frac{1}{y_t} \tag{A.70}$$

where β is the discount factor.

$$\frac{\delta L}{\delta b_t} = \lambda_{f_{t+1}} R_t - \lambda_{f_t} [1 + \kappa[\frac{b_t}{y_t} - \bar{b}]] \tag{A.71}$$

At optimum it is equal to zero,

$$\frac{\lambda_{f_{t+1}}}{\lambda_{f_t}} = \frac{[1 + \kappa[\frac{b_t}{y_t} - \bar{b}]]}{R_t} \tag{A.72}$$

Considering A.14 and and A.17,

$$\frac{[c_{f_{t+1}} + c_{i_{t+1}} - \psi_1 l_{i_{t+1}}^{v_1} - \psi_2 l_{f_{t+1}}^{v_2}]^{-\sigma}}{[c_{f_t} + c_{i_t} - \psi_1 l_{i_t}^{v_1} - \psi_2 l_{f_t}^{v_2}]^{-\sigma}} = \frac{[1 + \kappa[\frac{b_t}{y_t} - \bar{b}]]}{R_t} \tag{A.73}$$

Following Neumeyer and Perri (2005) and Tiryaki (2012), we will disregard the impact of bonds given κ tends to be negligible.

A.2.6 Model: Firms

Production function(CRS) is given by:
Formal output:

$$y_{f_t} = A_{f_t} {l_{f_t}}^{(1-\alpha)} k_t^{1-\alpha} \tag{A.74}$$

Informal output:

$$y_{i_t} = A_{i_t} {l_{i_t}}^{\beta} \tag{A.75}$$

Firms choose formal labour, informal labour and capital with the goal of maximising profit given by:

$$y_{f_t} + y_{i_t} - (1 - \theta) l_{f_t} w_{f_t} - R_{t-1} \theta l_{f_t} w_{f_t} - l_{i_t} w_{i_t} - r_t k_{t-1} \tag{A.76}$$

where θ is the working capital parameter.

A.2.7 Profit Equation

Profit equation:

$$\Pi_t = y_{f_t} + y_{i_t} - (1-\theta) l_{f_t} w_{f_t} - R_{t-1} \theta l_{f_t} w_{f_t} - l_{i_t} w_{i_t} - r_t k_{t-1} \tag{A.77}$$

A.2.8 Labour- Firm Side

From A.26, we get

$$\Pi_t = y_{f_t} + y_{i_t} - (1-\theta) l_{f_t} w_{f_t} - R_{t-1} \theta l_{f_t} w_{f_t} - l_{i_t} w_{i_t} - r_t k_t$$

Using A.8 and A.6,

$$\frac{\delta \Pi_t}{\delta l_{f_t}} = A_{f_t} \alpha {l_{f_t}}^{(\alpha-1)} k_{t-1}^{(1-\alpha)} - A_{i_t} (\beta) {l_{i_t}}^{-\beta-1} + [-1 + \theta(1 - R_{t-1}) + \frac{1}{\mu_i}] w_{f_t} \tag{A.78}$$

At optimum, the derivative equals to zero.
FOC:

$$A_{f_t} \alpha {l_{f_t}}^{(\alpha-1)} k_{t-1}^{(1-\alpha)} - A_{i_t} (\beta) {l_{i_t}}^{-\beta-1} = [1 - \theta(1 - R_{t-1}) - \frac{1}{\mu_i}] w_{f_t} \tag{A.79}$$

$$\frac{A_{f_t} \alpha {l_{f_t}}^{(\alpha-1)} k_{t-1}^{(1-\alpha)} - A_{i_t} (\beta) {l_{i_t}}^{-\beta-1}}{[1 - \theta(1 - R_{t-1}) - \frac{1}{\mu_i}]} = w_{f_t} \tag{A.80}$$

A.2.9 Capital- Firm Side

Now from equation A.26,

$$\frac{\delta \Pi_t}{\delta k_{t-1}} = A_{f_t} (1-\alpha) k_{t-1}^{-\alpha} [l_{f_t}]^{\alpha} - r_t \tag{A.81}$$

First order condition:

$$A_{f_t} (1-\alpha) k_{t-1}^{-\alpha} [l_{f_t}]^{\alpha} = r_t \tag{A.82}$$

A.2.10 Labour- Equilibrium and Log Linearization

From A.11 and A.29,

$$\frac{A_t\alpha {l_{f_t}}^{(\alpha-1)}k_{t-1}^{1-\alpha} - A_t\gamma_i\beta {l_{i_t}}^{\beta-1}k_{t-1}^{1-\beta}}{[1-\theta(1-R_{t-1})-\frac{1}{\mu}]} = \frac{-\psi_1 v_1 {l_{i_t}}^{v_1-1} + \psi_2 v_2 l_{f_t}^{v_2-1}}{(1-\tau-\frac{1}{\mu})} \tag{A.83}$$

$$\begin{aligned}
&\frac{\tilde{A}_f\alpha\tilde{k}^{1-\alpha}\tilde{l}_f^{\alpha-1}}{\tilde{A}_f\alpha\tilde{l}_f^{(\alpha-1)}\tilde{k}^{(1-\alpha)} - \tilde{A}_i\beta\tilde{l}_i^{\beta-1}\tilde{k}^{\beta-1}}\hat{A}_{f_t} - \frac{\tilde{A}_i\beta\tilde{l}_i^{-\beta-1}}{\tilde{A}_f\alpha\tilde{l}_f^{(\alpha-1)}\tilde{k}^{(1-\alpha)} - \tilde{A}_i\beta\tilde{l}_i^{\beta-1}\tilde{k}^{\beta-1}}\hat{A}_{i_t} \\
&+\frac{\tilde{A}_f\alpha(\alpha-1)\tilde{k}^{1-\alpha}\tilde{l}_f^{\alpha-2} + \tilde{A}_i\beta(\beta-1)\tilde{l}_i^{\beta-2}k^{1-\beta}}{\tilde{A}_f\alpha\tilde{l}_f^{(\alpha-1)}\tilde{k}^{(1-\alpha)} - \tilde{A}_i\beta\tilde{l}_i^{\beta-1}\tilde{k}^{\beta-1}}\hat{l}_{f_t} \\
&= \frac{\theta\hat{R}_{t-1}}{1-\theta(1-\bar{R})-\frac{1}{\mu_i}} + \frac{\psi_2 v_2(v_2-1)\tilde{l}_f^{v_2-2} + \psi_1 v_1(v_1-1)(\tilde{l}_i^{v_1-2}}{-\psi_1 v_1\tilde{l}_i^{v_1-1} + \psi_2 v_2(\tilde{l}_f)^{v_2-1}}\hat{l}_{f_t}
\end{aligned} \tag{A.84}$$

From A.11 and A.28, we obtain,

$$\begin{aligned}
&\frac{\tilde{A}_f\alpha\tilde{k}^{1-\alpha}\tilde{l}_f^{\alpha-1}}{(1-\theta(1-\bar{R})-\frac{1}{\mu_i})\tilde{w}_f}\hat{A}_{f_t} - \frac{\tilde{A}_i\beta\tilde{l}_i^{-\beta-1}}{(1-\theta(1-\bar{R})-\frac{1}{\mu_i})\tilde{w}_f}\hat{A}_{i_t} \\
&+\frac{\tilde{A}_f\alpha(\alpha-1)\tilde{k}^{1-\alpha}\tilde{l}_f^{\alpha-2} + \tilde{A}_i\beta(\beta-1)\tilde{l}_i^{\beta-2}k^{1-\beta}}{(1-\theta(1-\bar{R})-\frac{1}{\mu_i})\tilde{w}_f}\hat{l}_{f_t} = \\
&\frac{\theta\hat{R}_{t-1}}{1-\theta(1-\bar{R})-\frac{1}{\mu_i}} + \frac{\psi_2 v_2(v_2-1)\tilde{l}_f^{v_2-2} + \psi_1 v_1(v_1-1)\tilde{l}_i^{v_1-2}}{(1-\tau-\frac{1}{\mu_i})\tilde{w}_f}\hat{l}_{f_t}
\end{aligned} \tag{A.85}$$

A.2.11 Informal Output

From A.24, we get

$$y_{i_t} = A_{i_t}{l_{i_t}}^{\beta} \tag{A.86}$$

$$lny_{i_t} = lnA_{i_t} + \beta lnl_{i_t} \tag{A.87}$$

$$\hat{y}_{i_t} = \hat{A}_{i_t} + \beta\hat{l}_{i_t} \tag{A.88}$$

A.2.12 Informal Consumption

$$y_{i_t} = c_{i_t} \tag{A.89}$$

$$\hat{y}_{i_t} = \hat{c}_{i_t} \tag{A.90}$$

A.2.13 Investment

From A.4 and A.5,

$$x_t = k_t - (1-\delta)k_{t-1} + \frac{\phi}{2}k_{t-1}[\frac{(k_t - k_{t-1})}{k_{t-1}}]^2 \tag{A.91}$$

$$\hat{x}_t = \frac{\tilde{k}\hat{k}_t - (1-\delta)\tilde{k}\hat{k}_{t-1}}{\delta k} + \frac{\frac{\phi^2}{2\tilde{k}^2}(\tilde{k}-\tilde{k})^2\hat{k}_{t-1} + \frac{\phi^2}{\tilde{k}}(\tilde{k}-\tilde{k})[\hat{k}_t - \hat{k}_{t-1}]}{\delta k}$$

$$\hat{x}_t = \frac{\tilde{k}\hat{k}_t - (1-\delta)\tilde{k}\hat{k}_{t-1}}{\delta k} \tag{A.92}$$

A.2.14 Net Exports

$$NX_t = y_{f_t} - c_{f_t} - x_t - b_t - \frac{\kappa}{2}y_{f_t}(\frac{b_t}{y_{f_t}} - \bar{b})^2 \tag{A.93}$$

$$lnNX_t = ln(y_{f_t} - c_{f_t} - x_t - b_t - \frac{\kappa}{2}y_{f_t}(\frac{b_t}{y_{f_t}} - \bar{b})^2) \tag{A.94}$$

$$\hat{nx}_t = \frac{\tilde{y}_f[(1 - \frac{\kappa}{2}(\bar{b}-\bar{b})^2) - \frac{\kappa}{2}\tilde{y}_f 2(\bar{b}-\bar{b})\frac{1}{\tilde{y}_f^2}]}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{b}}\hat{y}_{f_t} - \frac{\tilde{c}_f}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{b}}\hat{c}_{f_t}$$

$$-\frac{\tilde{x}}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{b}}\hat{x}_t - \frac{\tilde{b}(1 + \frac{\kappa}{2}\tilde{y}_f 2(\bar{b}-\bar{b}))}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{b}}\hat{b}_t \tag{A.95}$$

$$\hat{nx}_t = \frac{\tilde{y}_f}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{b}}\hat{y}_{f_t} - \frac{\tilde{c}_f}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{b}}\hat{c}_{f_t} - \frac{\tilde{x}}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{b}}\hat{x}_t - \frac{\tilde{b}}{\tilde{y}_f - \tilde{c}_f - \tilde{x} - \tilde{b}}\hat{b}_t \tag{A.96}$$

A.2.15 Government

$$g_t = \tau l_{f_t} w_{f_t} \tag{A.97}$$

From A.10, we get,

$$\hat{g}_t = \hat{l}_{f_t} + \frac{\psi_2 v_2(v_2-1)\tilde{l}_f^{v_2-2} + \psi_1 v_1(v_1-1)\tilde{l}_i^{v_1-2}}{1 - \tau - \frac{1}{\mu}}\tilde{l}_f\hat{l}_{f_t} \tag{A.98}$$

A.2.16 Capital-Equilibrium and Log Linearization

Equating Equation A.22 with A.31, we get

$$\frac{R_{t-1}}{[1+\kappa[\frac{b_{t-1}}{y_{t-1}}-\bar{b}]]}=\frac{A_{f_t}(1-\alpha)k_{t-1}^{-\alpha}[l_{f_t}]^{\alpha}-1+\delta+\frac{1}{(k_{t-1})}(k_t-k_{t-1})+\frac{1}{2(k_{t-1}^2)}(k_t-k_{t-1})^2}{1+\frac{1}{(k_{t-2})}(k_{t-1}-k_{t-2})} \tag{A.99}$$

Following Neumeyer and Perri (2005) and Tiryaki (2012), we will disregard the impact of bonds given κ tends to be negligible.

Log Linearising,

$$\hat{R}_{t-1}-\kappa\bar{b}\hat{b}_{t-1}+[\frac{1}{\tilde{k}}\tilde{k}\hat{k}_{t-1}-\frac{\tilde{k}}{\tilde{k}^2}\tilde{k}\hat{k}_{t-2}]-\frac{\frac{(1-\alpha)\tilde{y}_f}{\tilde{k}}}{\frac{(1-\alpha)\tilde{y}_f}{\tilde{k}}-1+\delta}\hat{A}_{f_t}$$

$$=\frac{(1-\alpha)(-\alpha)\frac{\tilde{y}_f}{\tilde{k}}}{(1-\alpha)\frac{\tilde{y}_f}{\tilde{k}}+(1-\delta)}\hat{k}_{t-1}-\frac{(1-\alpha)(-\alpha)\frac{\tilde{y}_f}{\tilde{k}}}{(1-\alpha)\frac{\tilde{y}_f}{\tilde{k}}+(1-\delta)}\hat{l}_{f_t}+\frac{[\frac{1}{\tilde{k}}\tilde{k}\hat{k}_t-\frac{\tilde{k}}{\tilde{k}^2}\tilde{k}\hat{k}_{t-1}]}{(1-\alpha)\frac{\tilde{y}_f}{\tilde{k}}+(1-\delta)}+(\tilde{k}-\tilde{k})\frac{[\frac{1}{\tilde{k}}\tilde{k}\hat{k}_t-\frac{\tilde{k}}{\tilde{k}^2}\tilde{k}\hat{k}_{t-1}]}{(1-\alpha)\frac{\tilde{y}_f}{\tilde{k}}+(1-\delta)}$$

$$-\frac{\frac{(1-\alpha)\tilde{y}_f}{\tilde{k}}}{\frac{(1-\alpha)\tilde{y}_f}{\tilde{k}}-1+\delta}\hat{A}_{f_t}+\hat{R}_{t-1}-\kappa\bar{b}\hat{b}_{t-1}+[\hat{k}_{t-1}-\hat{k}_{t-2}]$$

$$=\frac{(1-\alpha)(-\alpha)\frac{\tilde{y}_f}{\tilde{k}}}{(1-\alpha)\frac{\tilde{y}_f}{\tilde{k}}+(1-\delta)}\hat{k}_{t-1}-\frac{(1-\alpha)(-\alpha)\frac{\tilde{y}_f}{\tilde{k}}}{(1-\alpha)\frac{\tilde{y}_f}{\tilde{k}}+(1-\delta)}\hat{l}_{f_t}+\frac{[\hat{k}_t-\hat{k}_{t-1}]}{(1-\alpha)\frac{\tilde{y}_f}{\tilde{k}}+(1-\delta)} \tag{A.100}$$

Following Neumeyer and Perri (2005) and Tiryaki (2012), we will disregard the impact of bonds given κ tends to be negligible.

A.2.17 Interest Rates

$$R_t=R_t^*D_t \tag{A.101}$$

$$lnR_t=lnR_t^*+lnD_t \tag{A.102}$$

$$\hat{R}_t=\hat{R}_t^*+\hat{D}_t \tag{A.103}$$

A.2.18 Foreign Interest Rate Shocks

$$\hat{R}_t^*=\rho_{R^*}\hat{R}_{t-1}^*+\epsilon_{R^*} \tag{A.104}$$

A.2.19 Independent Country Risk Shocks

$$\hat{D}_t = \rho_D \hat{D}_{t-1} + \epsilon_D \tag{A.105}$$

A.2.20 Induced Country Risk Shocks

$$\hat{D}_t = \bar{\eta}_f (E_t \hat{A}_{f_{(t+1)}}) + \epsilon_{\eta_{ft}} \tag{A.106}$$

$$\hat{D}_t = \bar{\eta}_i (E_t \hat{A}_{f_{(t+1)}}) + \epsilon_{\eta_{it}} \tag{A.107}$$

A.3 Structural VAR Models

A.3.1 Model 2

Let us consider the BVAR(4) with 4 variables: Country risk of India, Indian real interest rate, government spending and informal output.

$$D_{20}Z_t = c_{21}{}^* + D_{21}Y_{t-1} + D_{22}Y_{t-2} + D_{23}Y_{t-3} + D_{24}Y_{t-4} + \nu_{2_t} \tag{A.108}$$

where $Y_t = (CR_t, INT_t, GOVT_t, INF_t)'$, where $CR_t, INT_t, GOVT_t, INF_t$ are the cyclical decomposition of HP filtered values for country risk of India, Indian real interest rate, government spending and informal output of India.

If we pre-multiply the model with D_{20}^{-1}, we get

$$Z_t = c_{21} + C_{21}Y_{t-1} + C_{22}Y_{t-2} + C_{23}Y_{t-3} + C_{24}Y_{t-4} + e_{2_t} \tag{A.109}$$

where $c_{21} = D_{20}^{-1}c_{21}^*$, $c_{2S} = D_0^{-1}D_s$ where s runs from 1 to 4 , and $e_{2_t} = D_{20}^{-1}\nu_{2_t}$

We identify D_0 matrix as that will help us extract the structural shocks from $\nu_{2_t} = D_{20}e_{2_t}$.

A.3.2 Model 3

Let us consider the BVAR(4) with 4 variables: formal productivity, country risk, Indian real interest rate, government spending and informal output.

$$F_{20}X_t = c_{22}^* + F_{21}X_{t-1} + F_{22}X_{t-2} + F_{23}X_{t-3} + +F_{24}X_{t-4} + \mu_{3_t} \tag{A.110}$$

where $X_t = (FORAP_t, CR_t, INT_t, GOVT_t, INF_t)'$, where $FORAP_t, CR_t, INT_t, GOVT_t, INF_t$ are the cyclical decomposition of HP filtered values for formal productivity shocks, country risk, Indian real interest rate, government spending and informal output of India.

If we pre-multiply the model with F_{20}^{-1}, we get

$$Z_t = c_{22} + G_{21}Y_{t-1} + G_{22}Y_{t-2} + G_{23}Y_{t-3} + G_{24}Y_{t-4} + e_{3_t} \tag{A.111}$$

where $c_{22} = F_{20}^{-1}c_{22}^*$, $G_{2s} = F_{20}^{-1}F_{2s}$ where s runs from 1 to 4 , and $e_{3_t} = F_{20}^{-1}\nu_{3_t}$

We identify F_{20} matrix to obtain structural shocks ν_{3_t} from $\nu_{3_t} = F_{20}e_{3_t}$.

A.3.3 Model 4

Let us consider the BVAR(4) with 5 variables: informal productivity, country risk, government spending and informal output.

$$J_{20}W_t = c^*_{23} + J_{21}W_{t-1} + J_{22}W_{t-2} + J_{23}W_{t-3} + +J_{24}X_{t-4} + \nu_{4_t} \tag{A.112}$$

where $W_t = (INFAP_t, CR_t, INT_t, GOVT_t, INF_t)'$, where $INFAP_t, CR_t, GOVT_t, INF_t$ are the cyclical decomposition of HP filtered values for informal productivity of India, country risk of India, GDP of India, investment of India and labour of India.

If we pre-multiply the model with J_0^{-1}, we get

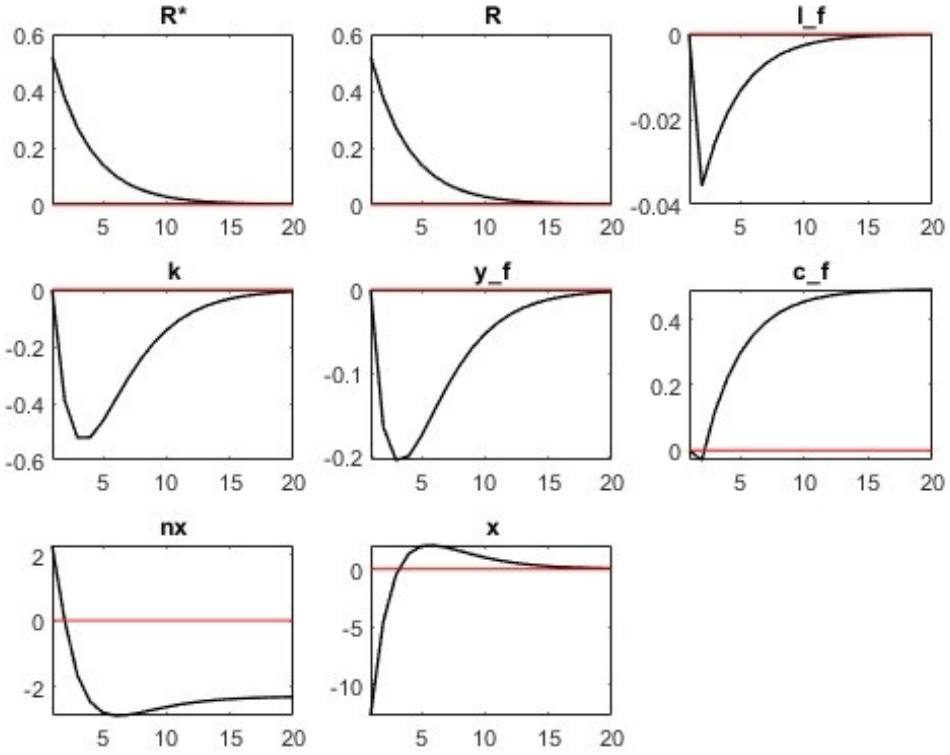
$$W_t = c_{23} + H_{21}W_{t-1} + H_{22}W_{t-2} + H_{23}W_{t-3} + H_{24}W_{t-4} + e_{4_t} \tag{A.113}$$

where $c_{23} = J_{20}^{-1}c^*_{23}$, $H_{2S} = J_0^{-1}J_{2s}$ where s runs from 1 to 4 , and $e_{4_t} = J_0^{-1}\nu_{4_t}$

We identify F_{20} matrix for structural shocks ν_{4_t} from $\nu_{4_t} = J_{20}e_{4_t}$.

A.4 Sensitivity Analysis

A.4.1 Tax Rate = 0.20



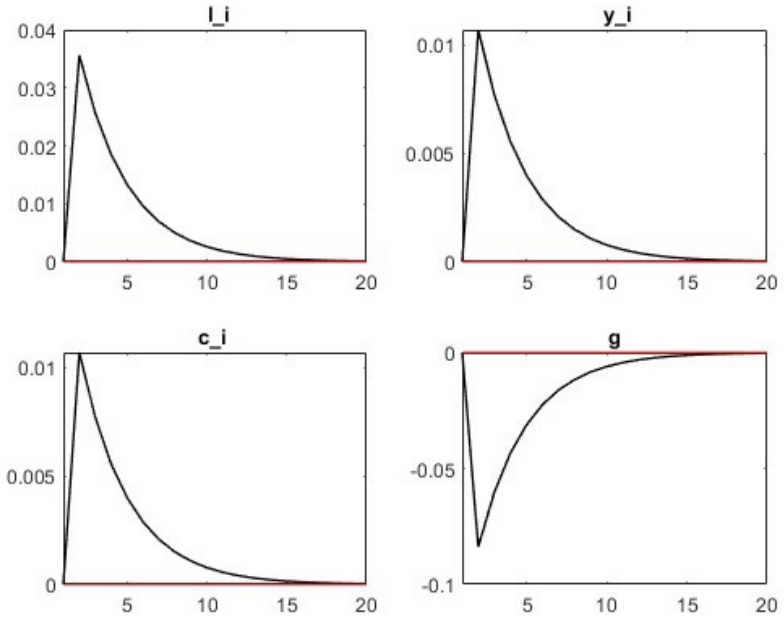
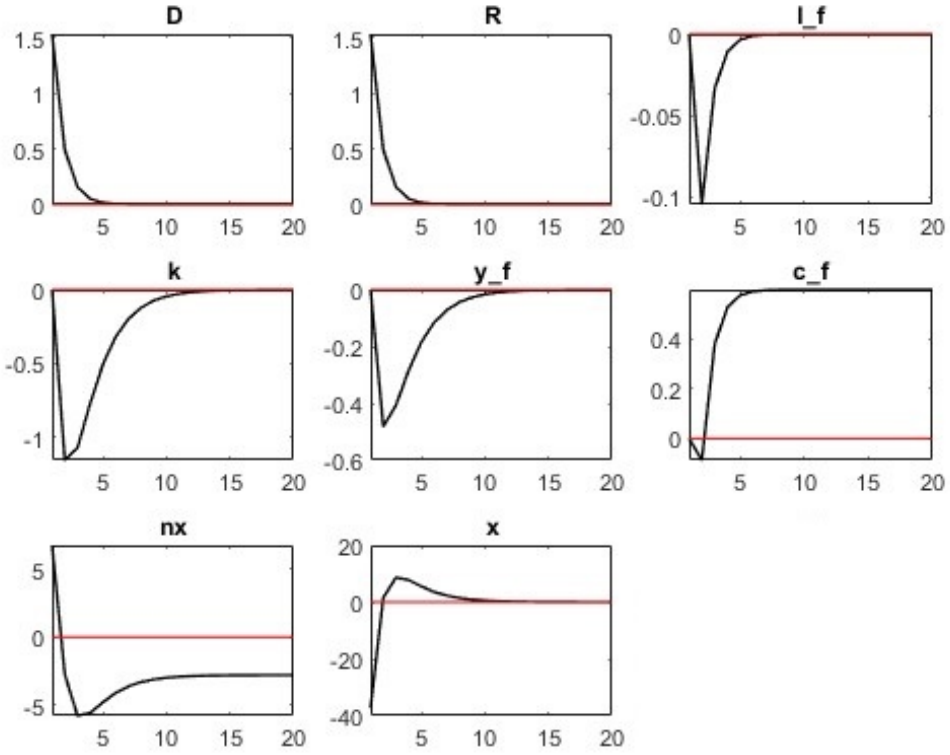


Figure A.1: Effect of Foreign Real Interest Rate Shocks (Tax Rate = 0.20)



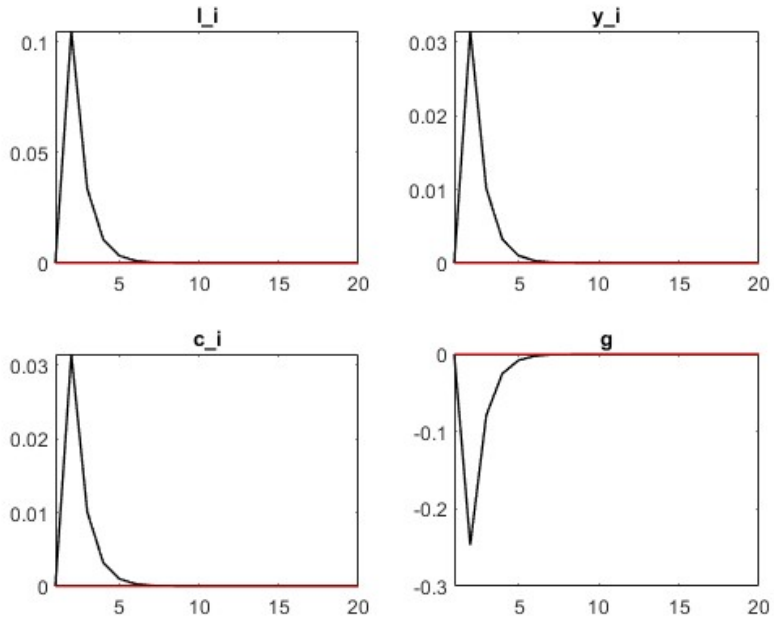


Figure A.2: Effect of Country Risk Shocks: Independent Case (Tax Rate = 0.20)

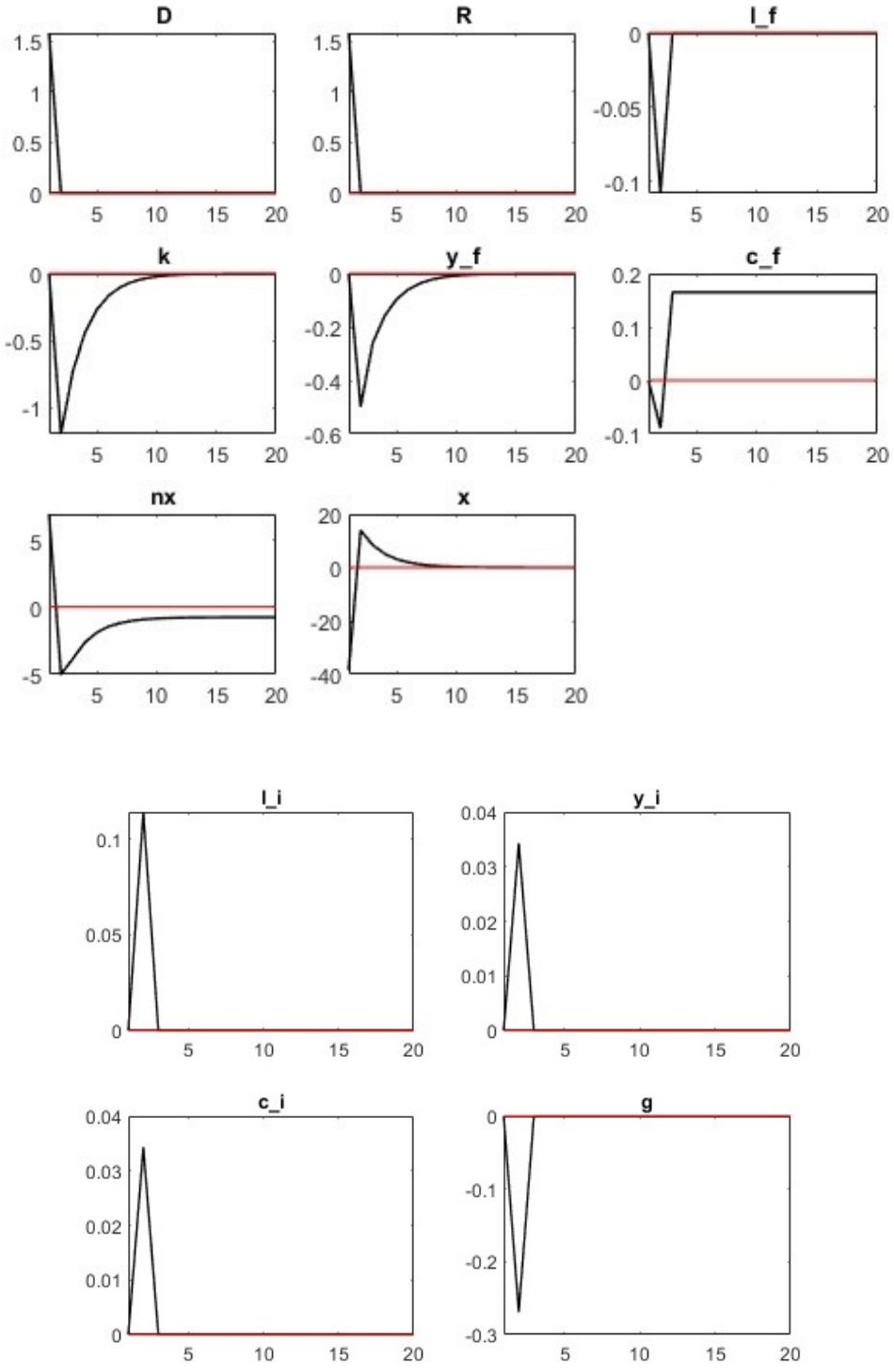


Figure A.3: Effect of Country Risk Shocks: Formal Productivity Induced Case (Tax Rate = 0.20)

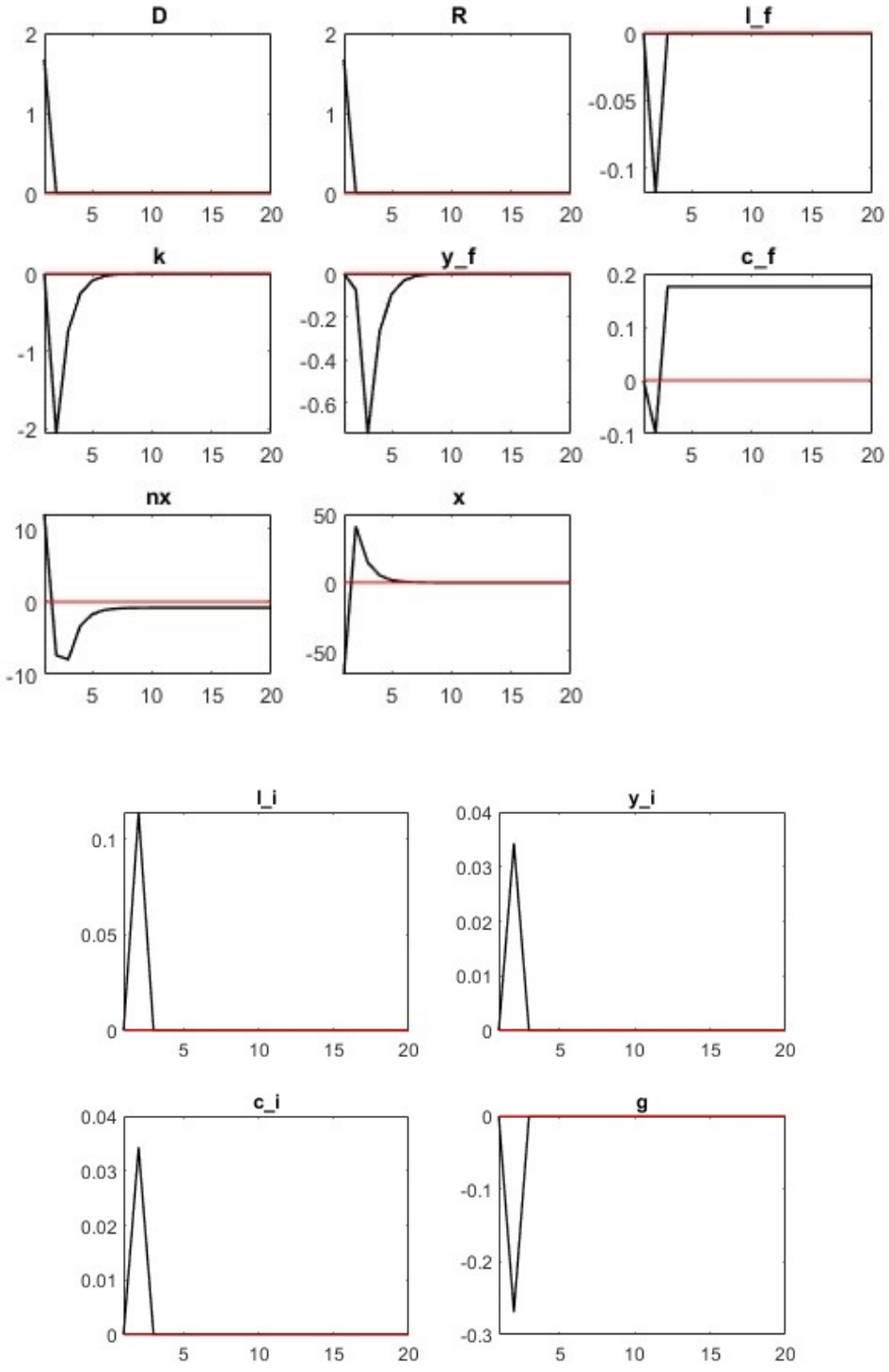


Figure A.4: Effect of Country Risk Shocks: Informal Productivity Induced Case (Tax Rate = 0.20)

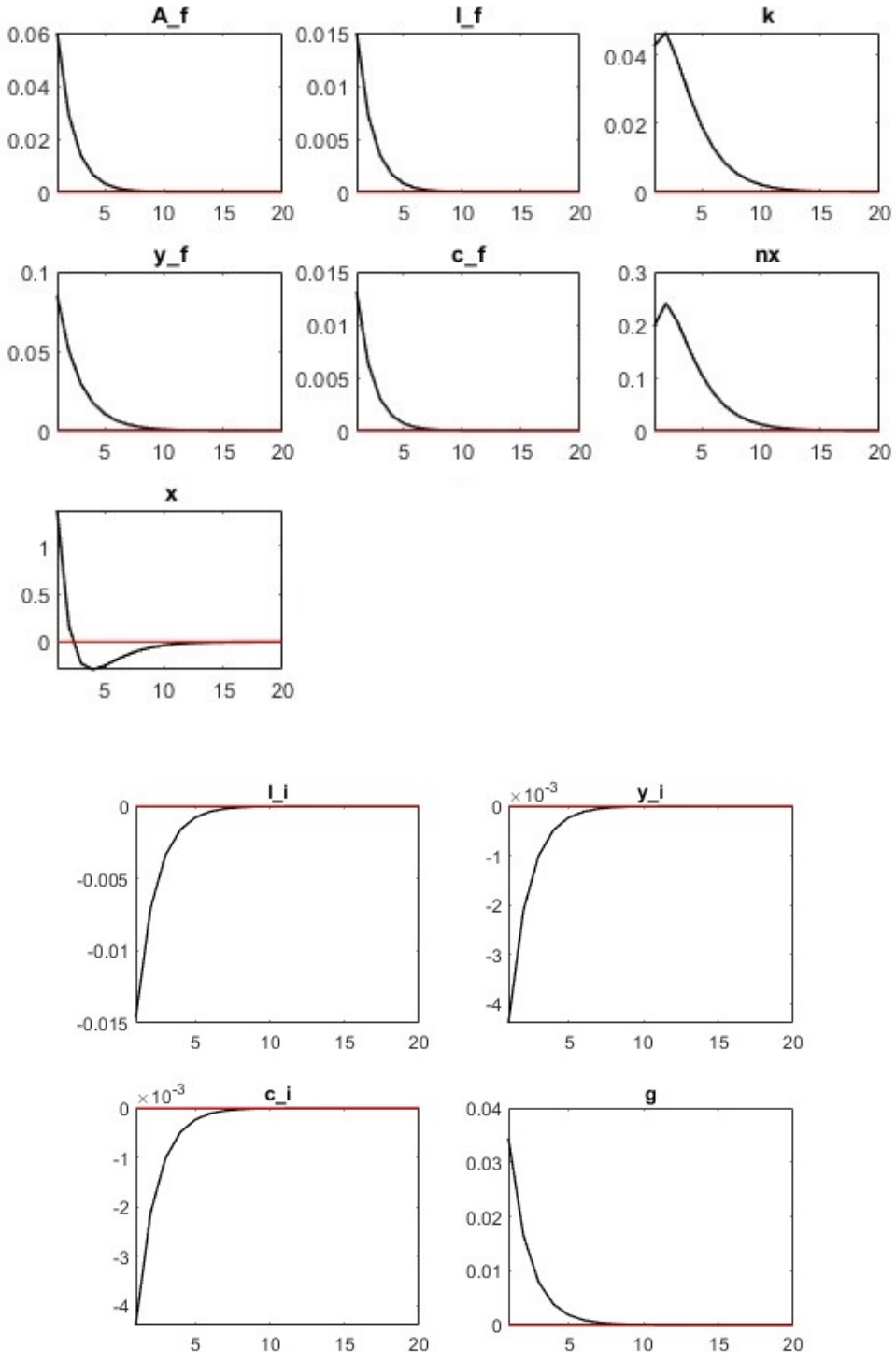


Figure A.5: Effect of Formal Productivity Shocks: Independent Case (Tax Rate = 0.20)

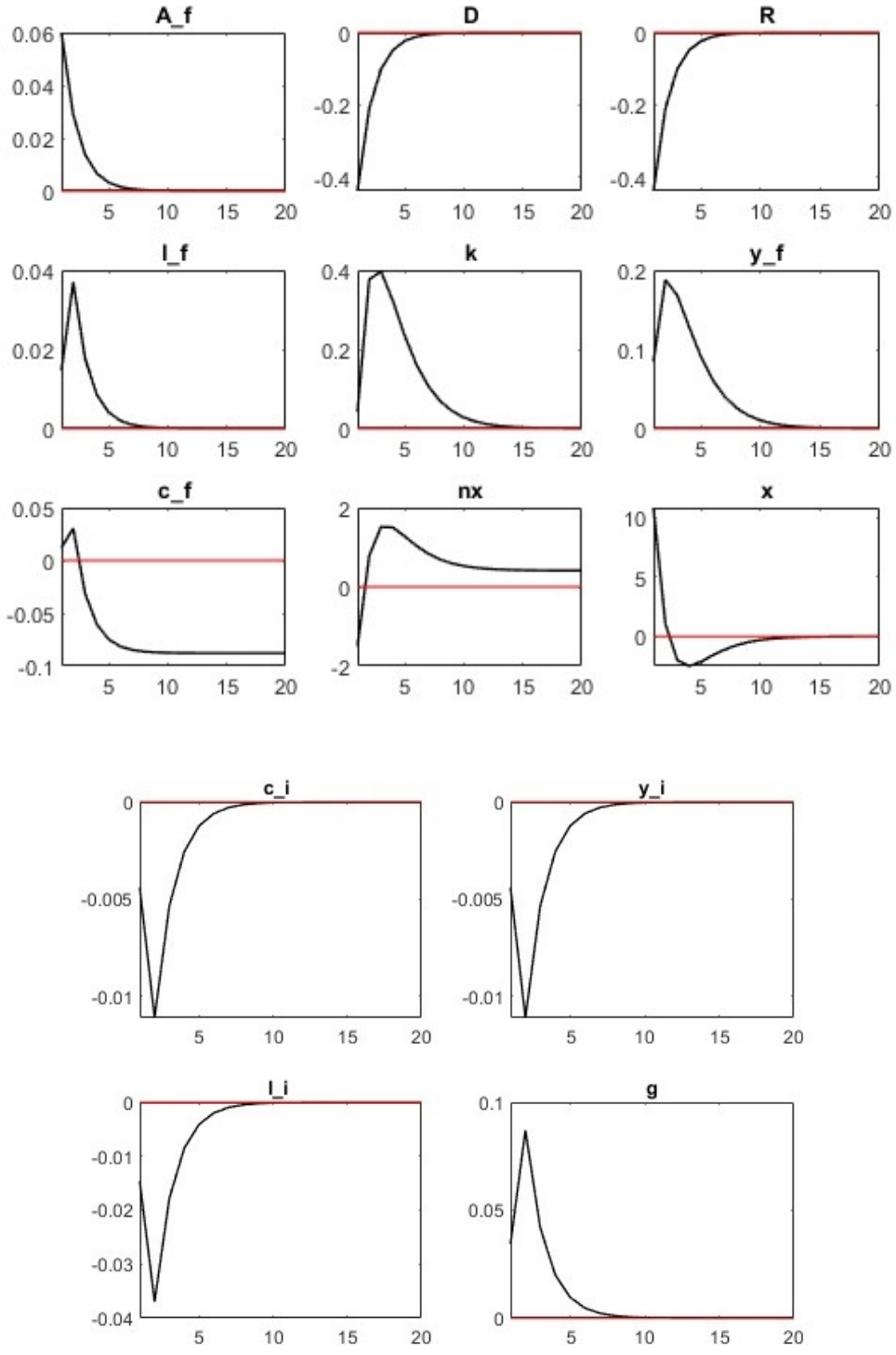


Figure A.6: Effect of Formal Productivity Shocks: Induced Case (Tax Rate = 0.20)

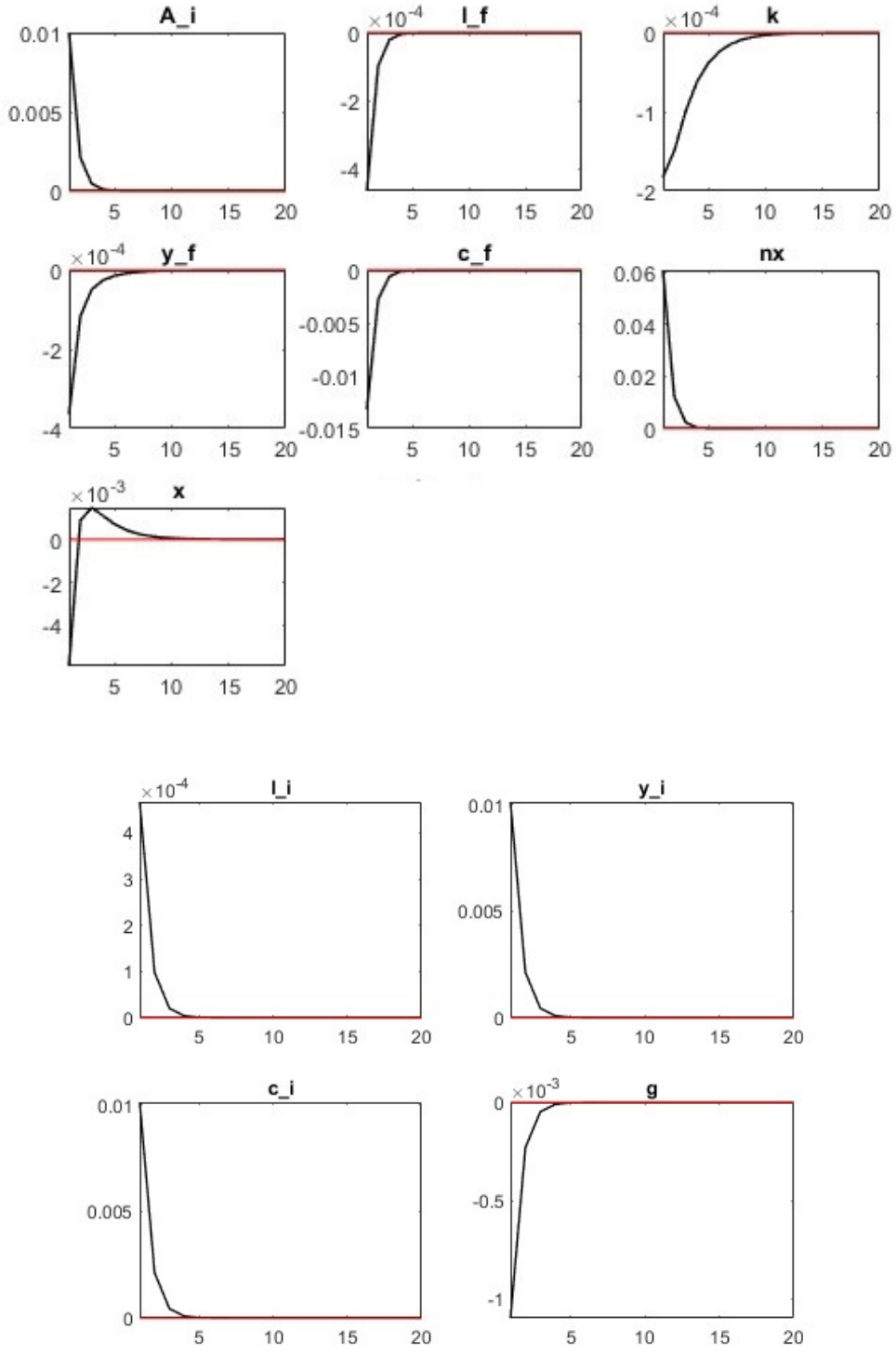


Figure A.7: Effect of Informal Productivity Shocks: Independent Case (Tax Rate = 0.20)

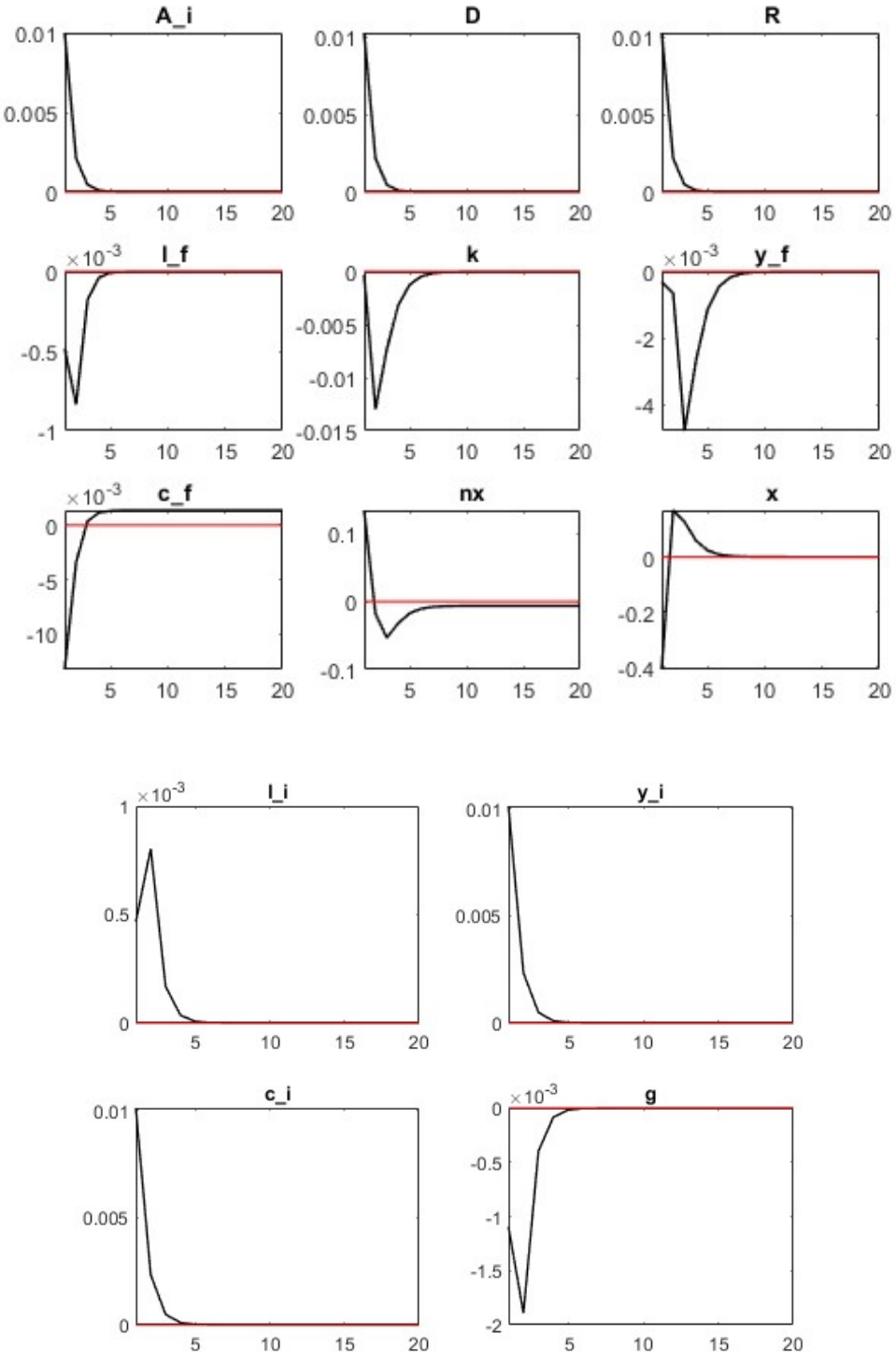


Figure A.8: Effect of Informal Productivity Shocks: Induced Case (Tax Rate = 0.20)

A.4.2 Wage Differential = 3

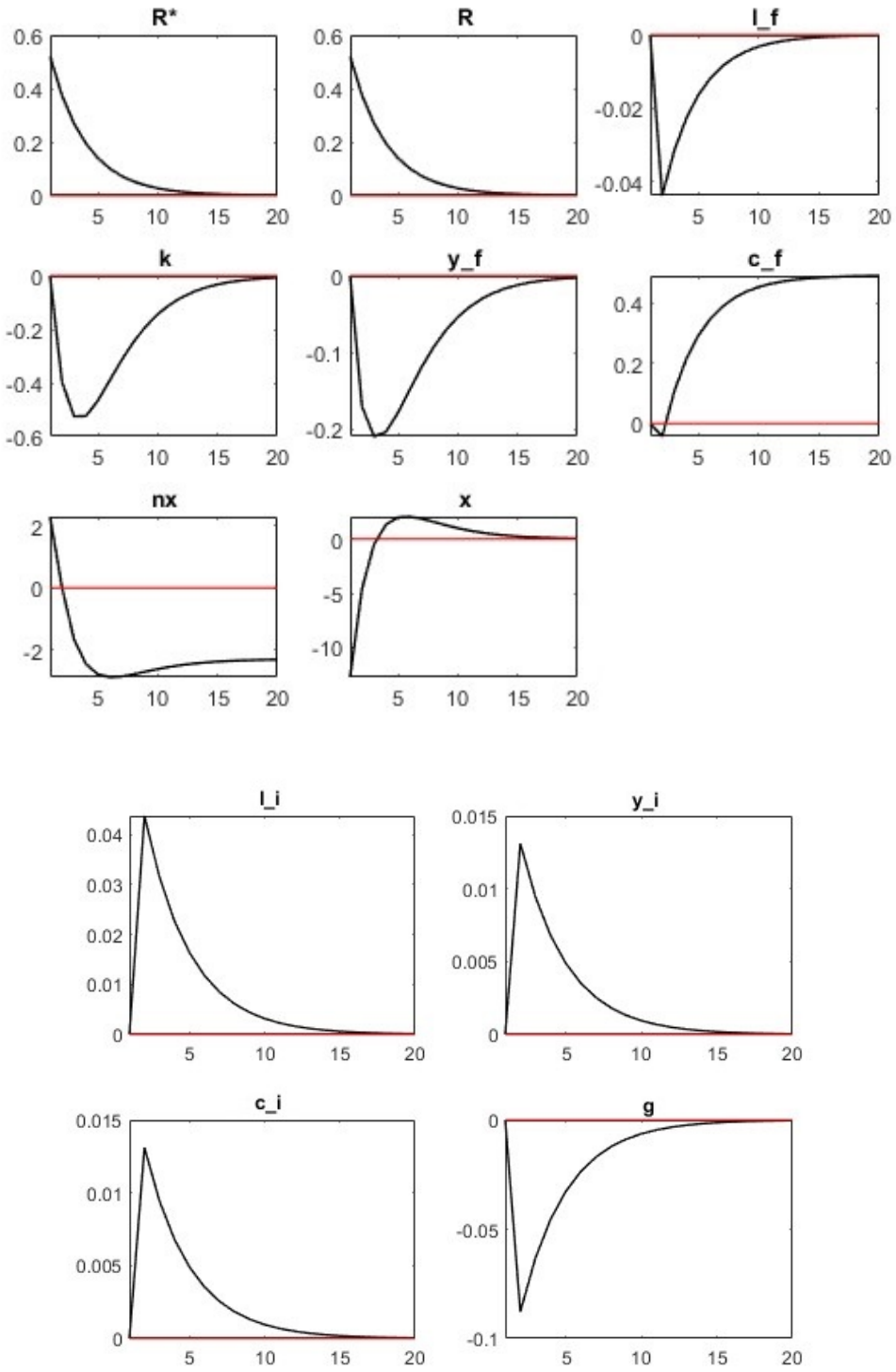


Figure A.9: Effect of Foreign Real Interest Rate Shocks (Wage Differential = 3)

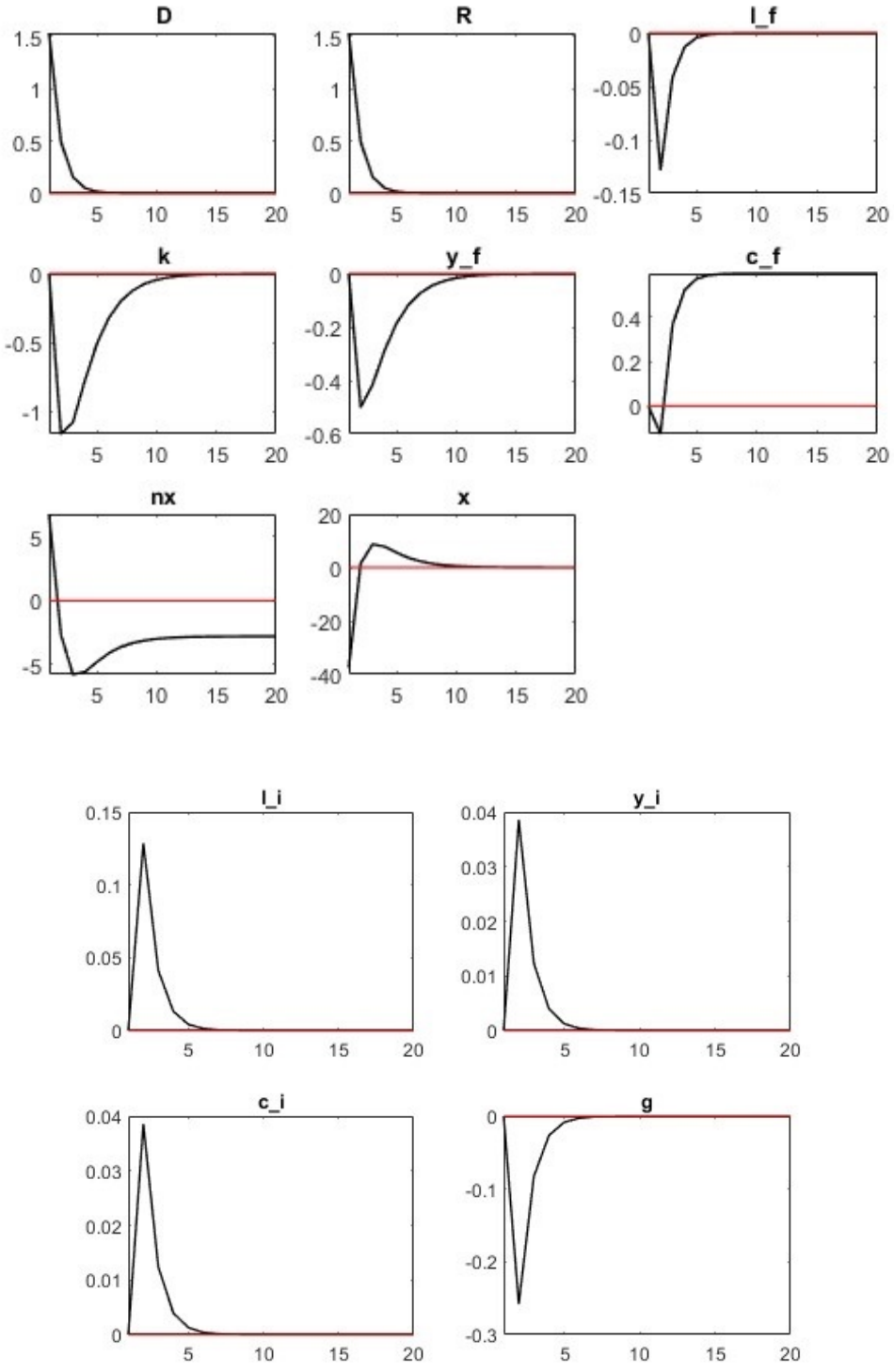


Figure A.10: Effect of Country Risk Shocks: Independent Case (Wage Differential = 3)

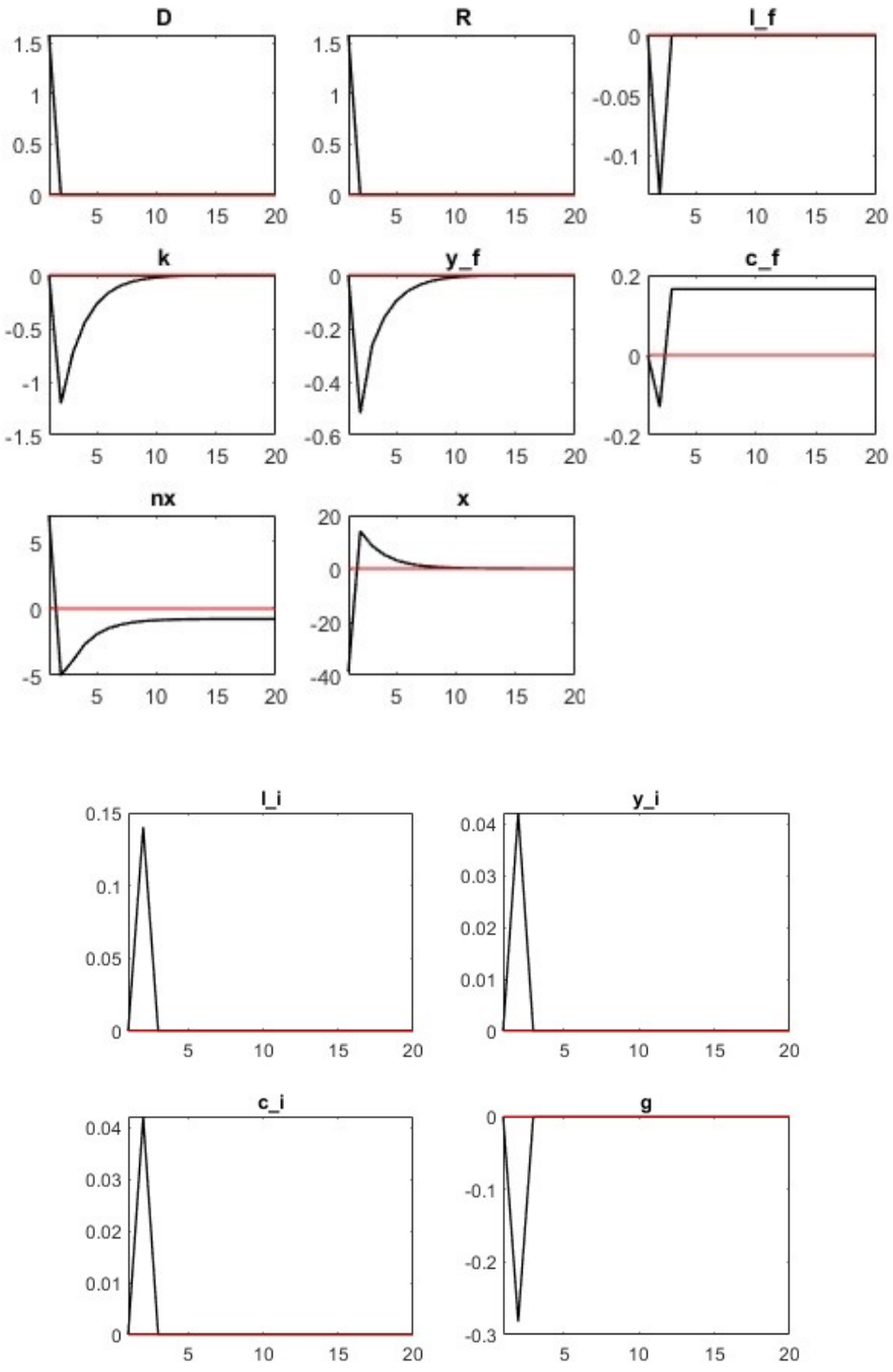


Figure A.11: Effect of Country Risk Shocks: Formal Productivity Induced Case (Wage Differential = 3)

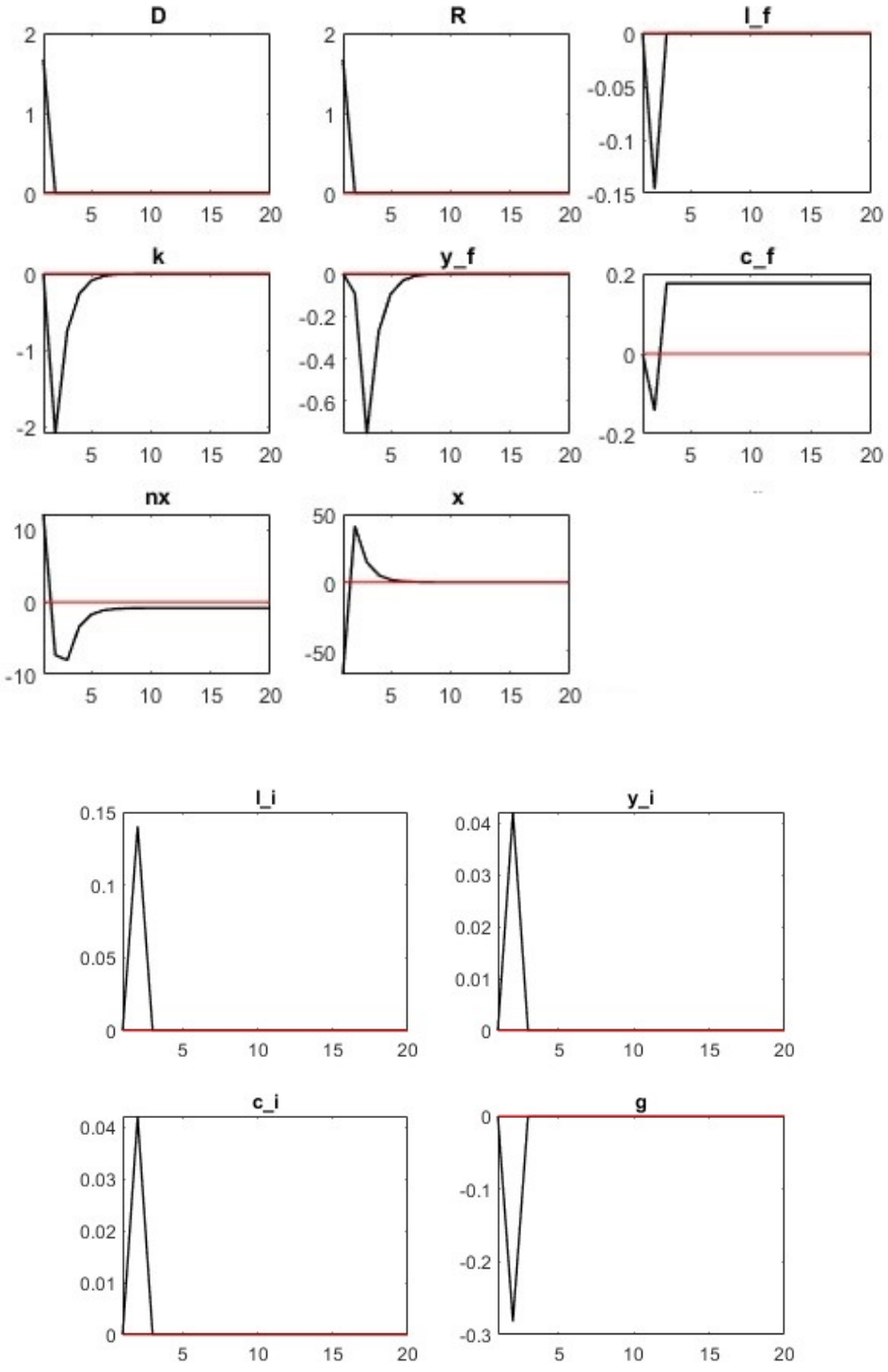


Figure A.12: Effect of Country Risk Shocks: Informal Productivity Induced Case (Wage Differential = 3)

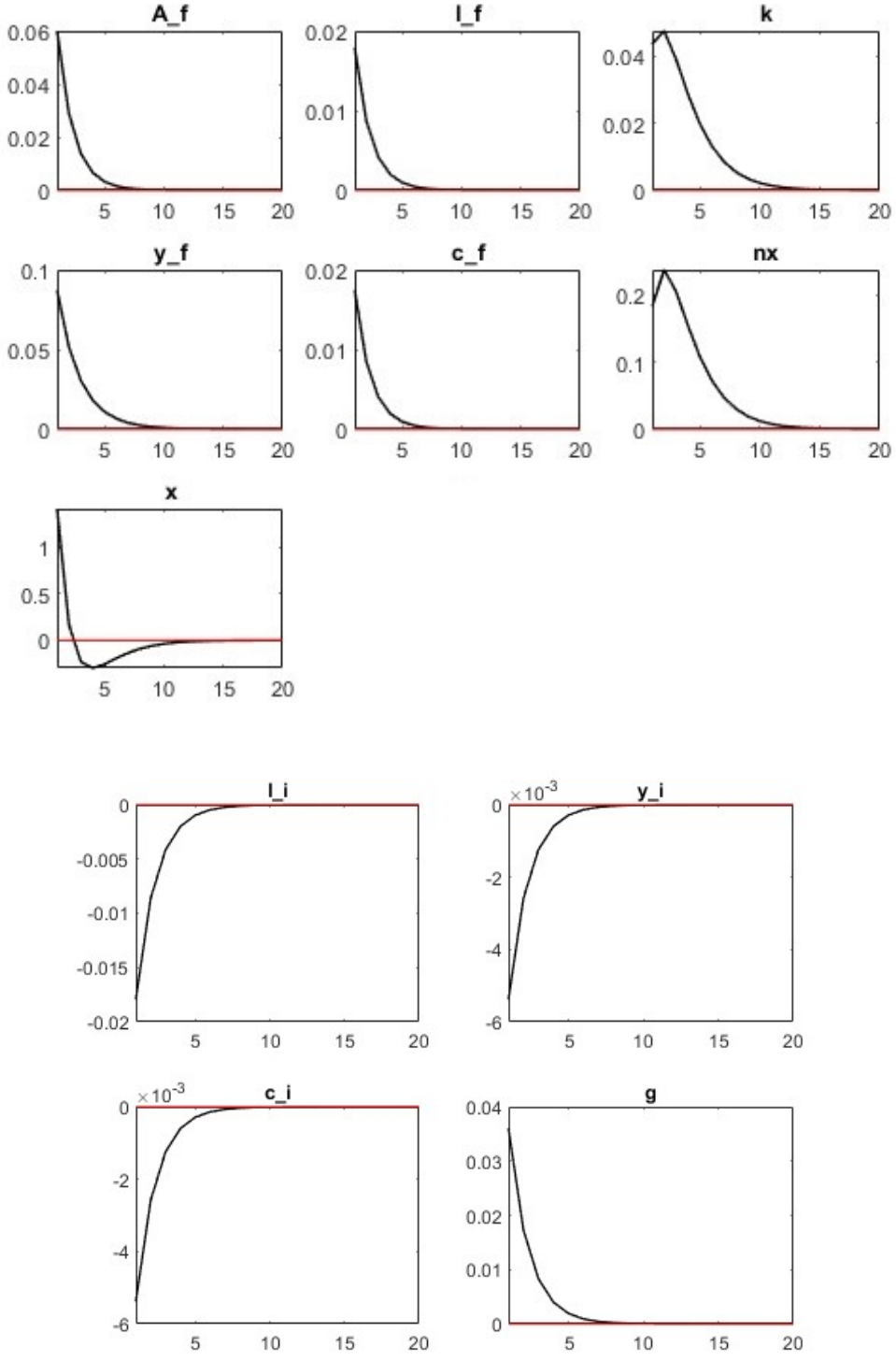


Figure A.13: Effect of Formal Productivity Shocks: Independent Case (Wage Differential = 3)

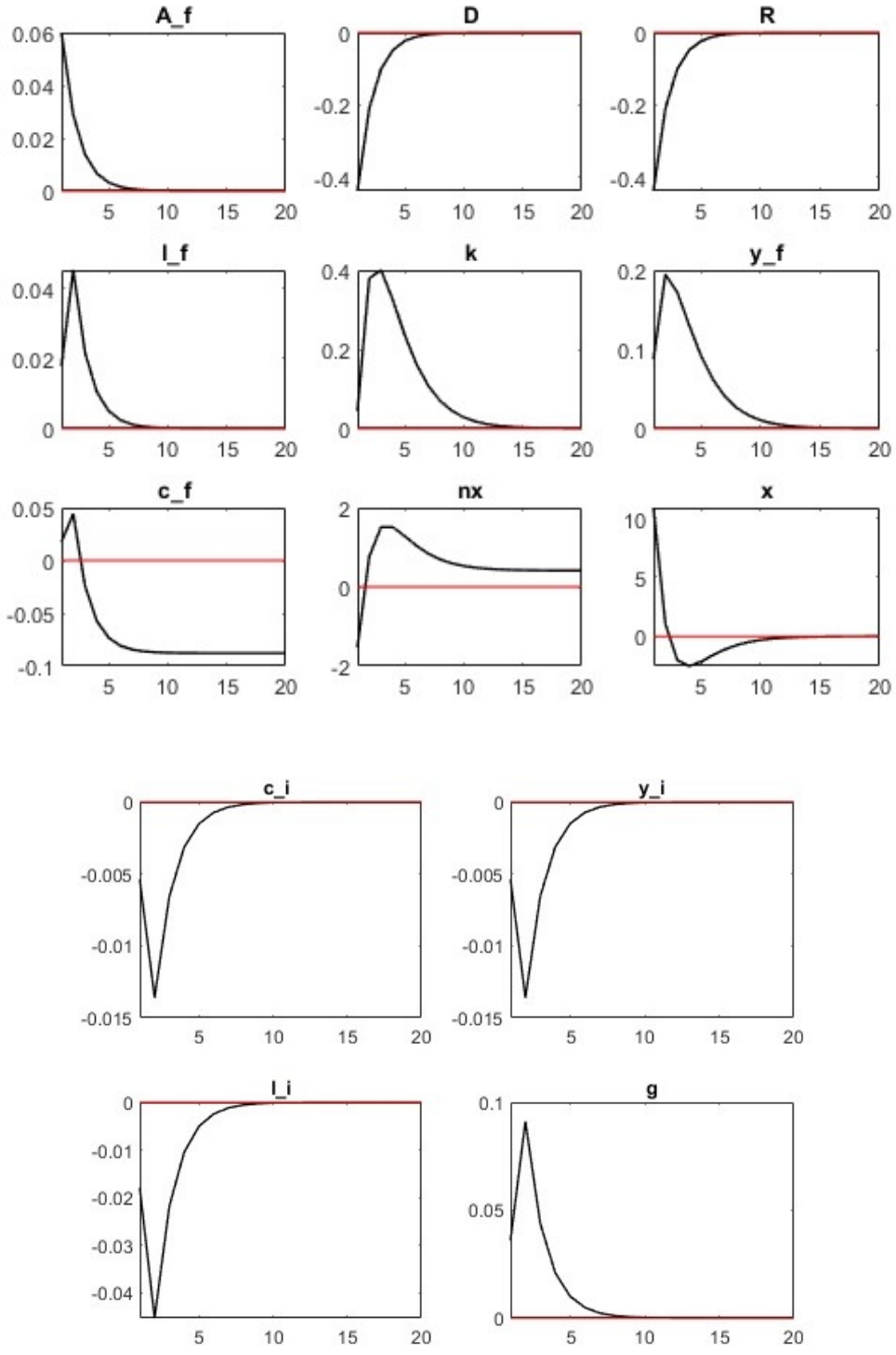


Figure A.14: Effect of Formal Productivity Shocks: Induced Case (Wage Differential = 3)

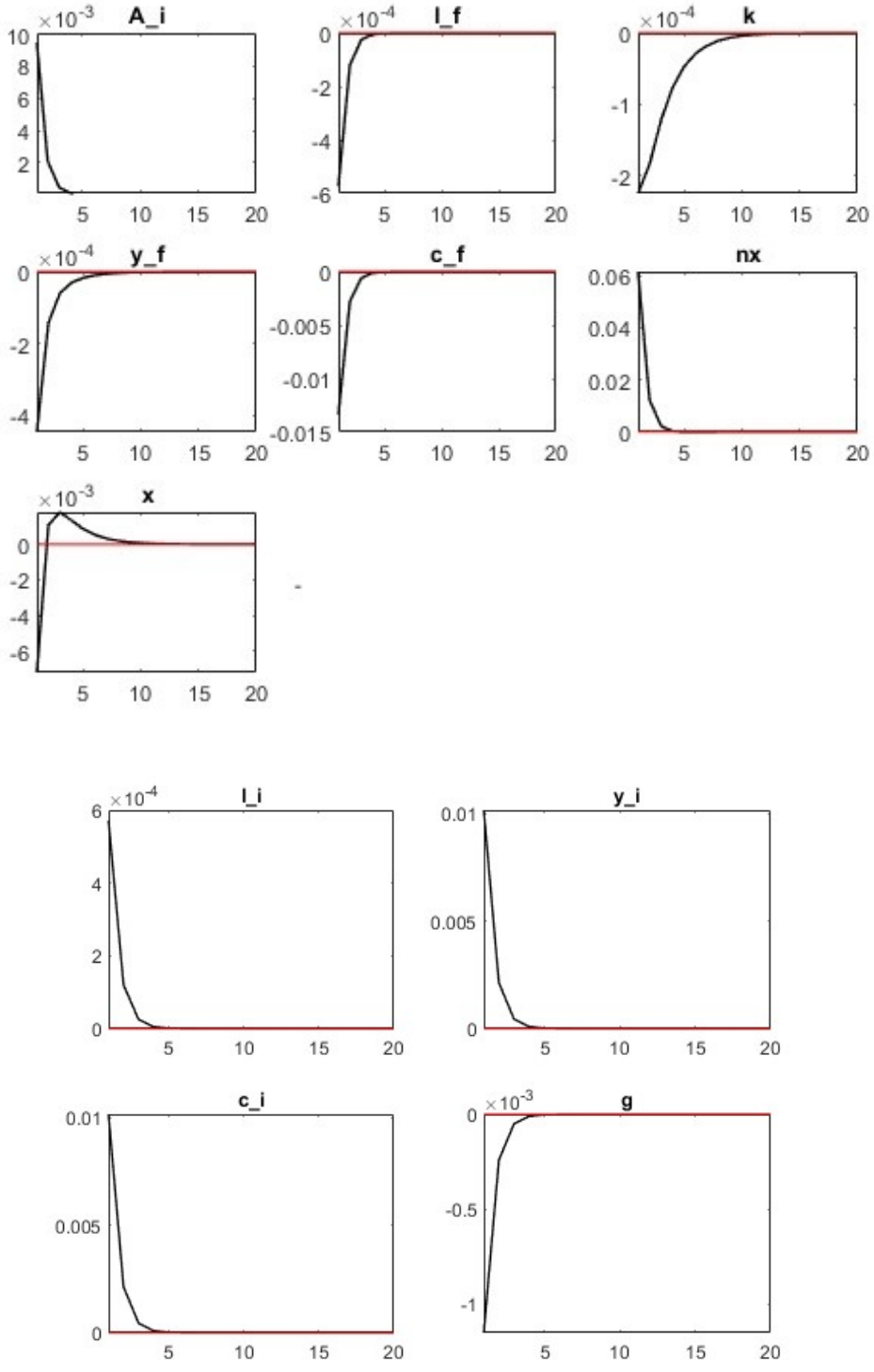


Figure A.15: Effect of Informal Productivity Shocks: Independent Case (Wage Differential = 3)

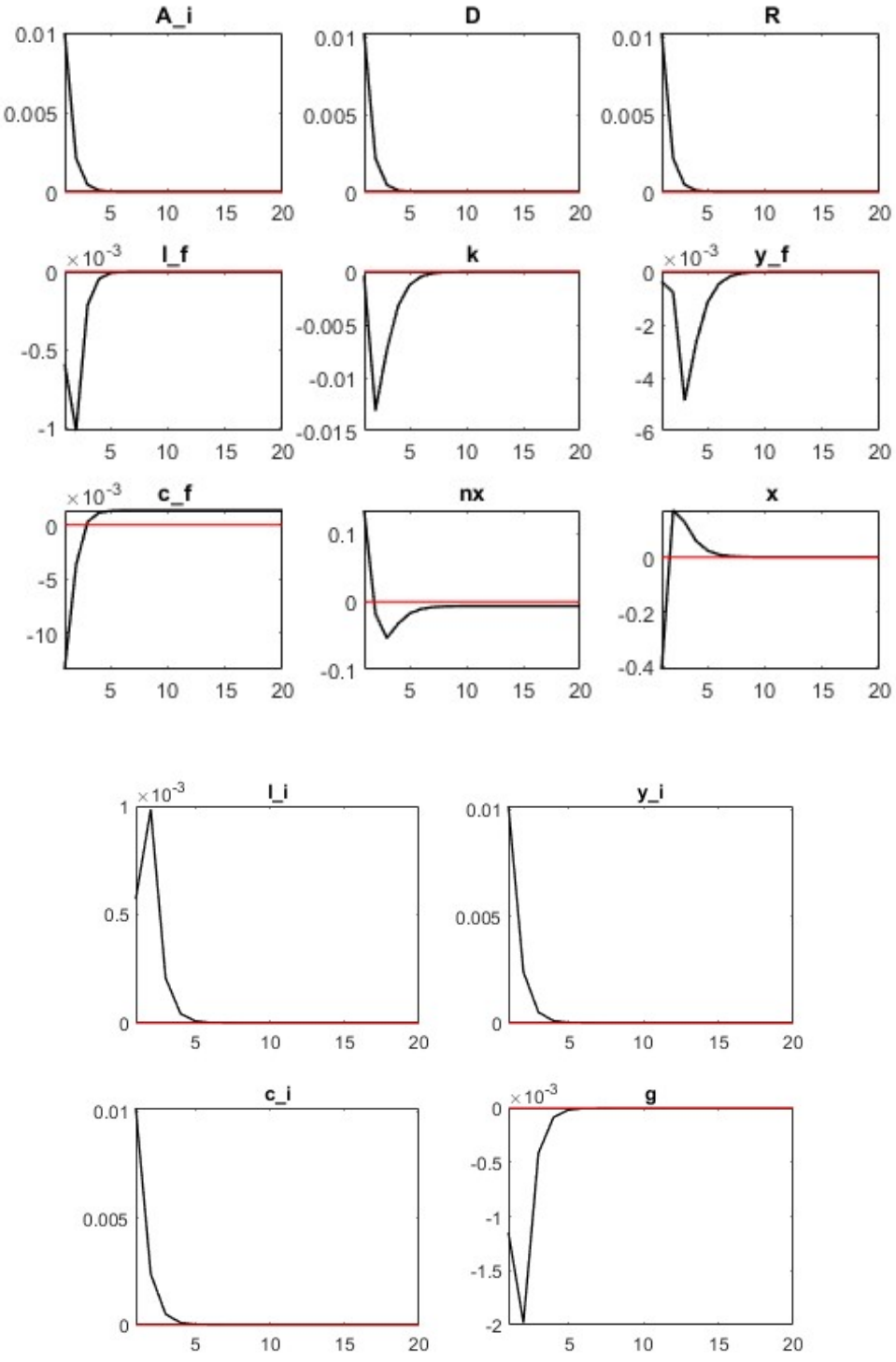


Figure A.16: Effect of Informal Productivity Shocks: Induced Case (Wage Differential = 3)

A.4.3 Working Capital = 1

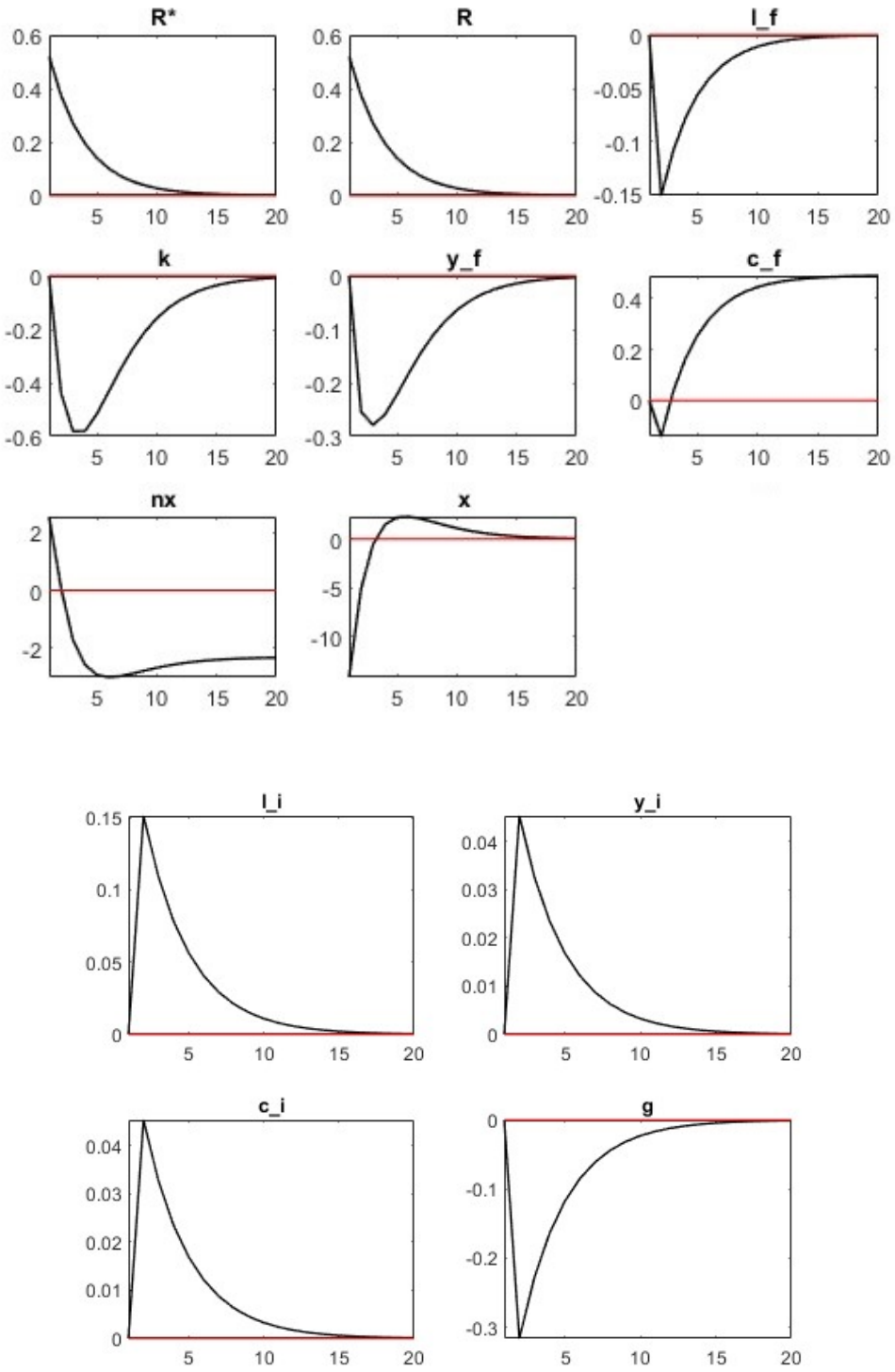


Figure A.17: Effect of Foreign Real Interest Rate Shocks (Working Capital = 1)

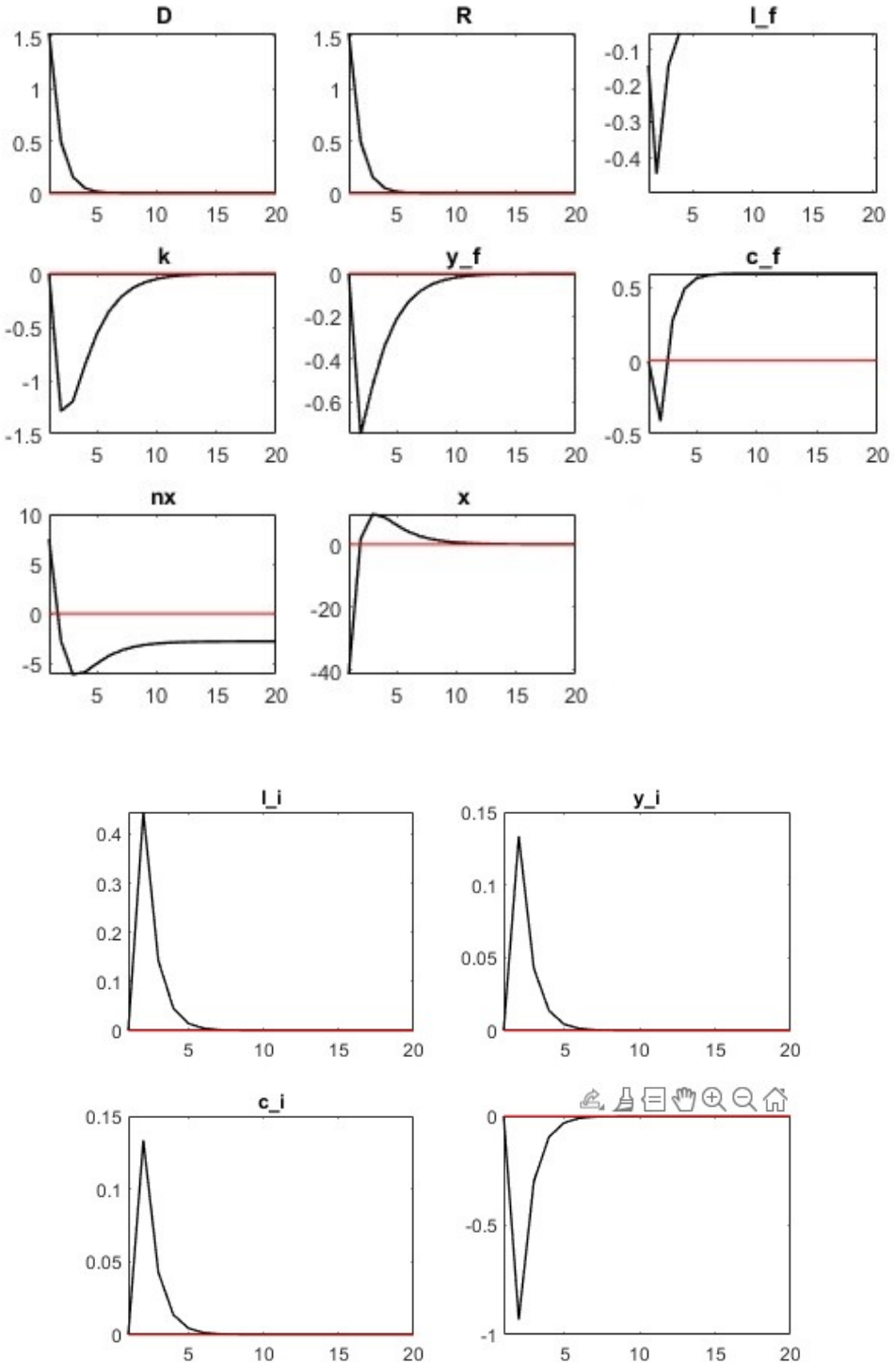


Figure A.18: Effect of Country Risk Shocks: Independent Case (Working Capital = 1)

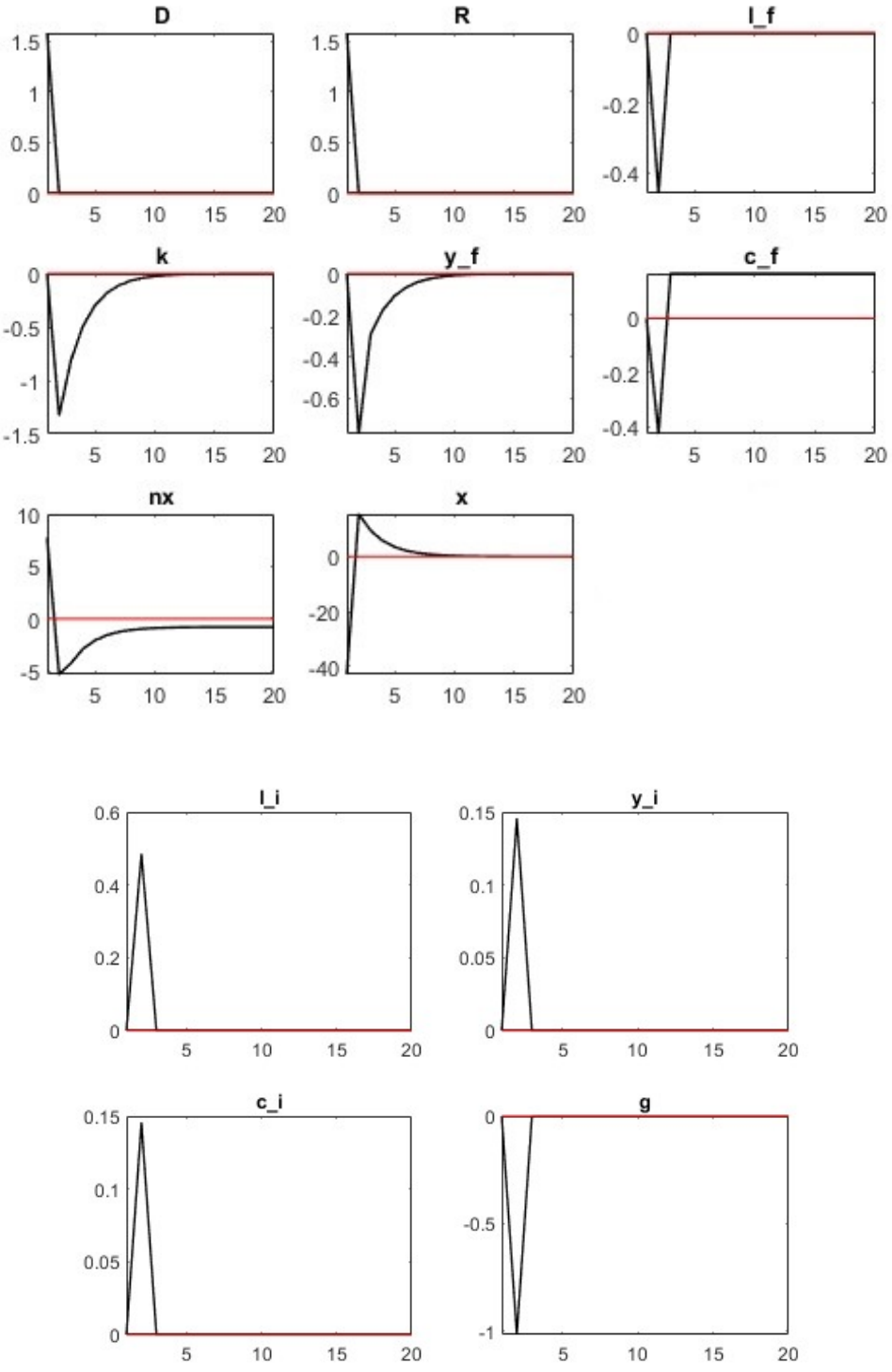


Figure A.19: Effect of Country Risk Shocks: Formal Productivity Induced Case (Working Capital = 1)

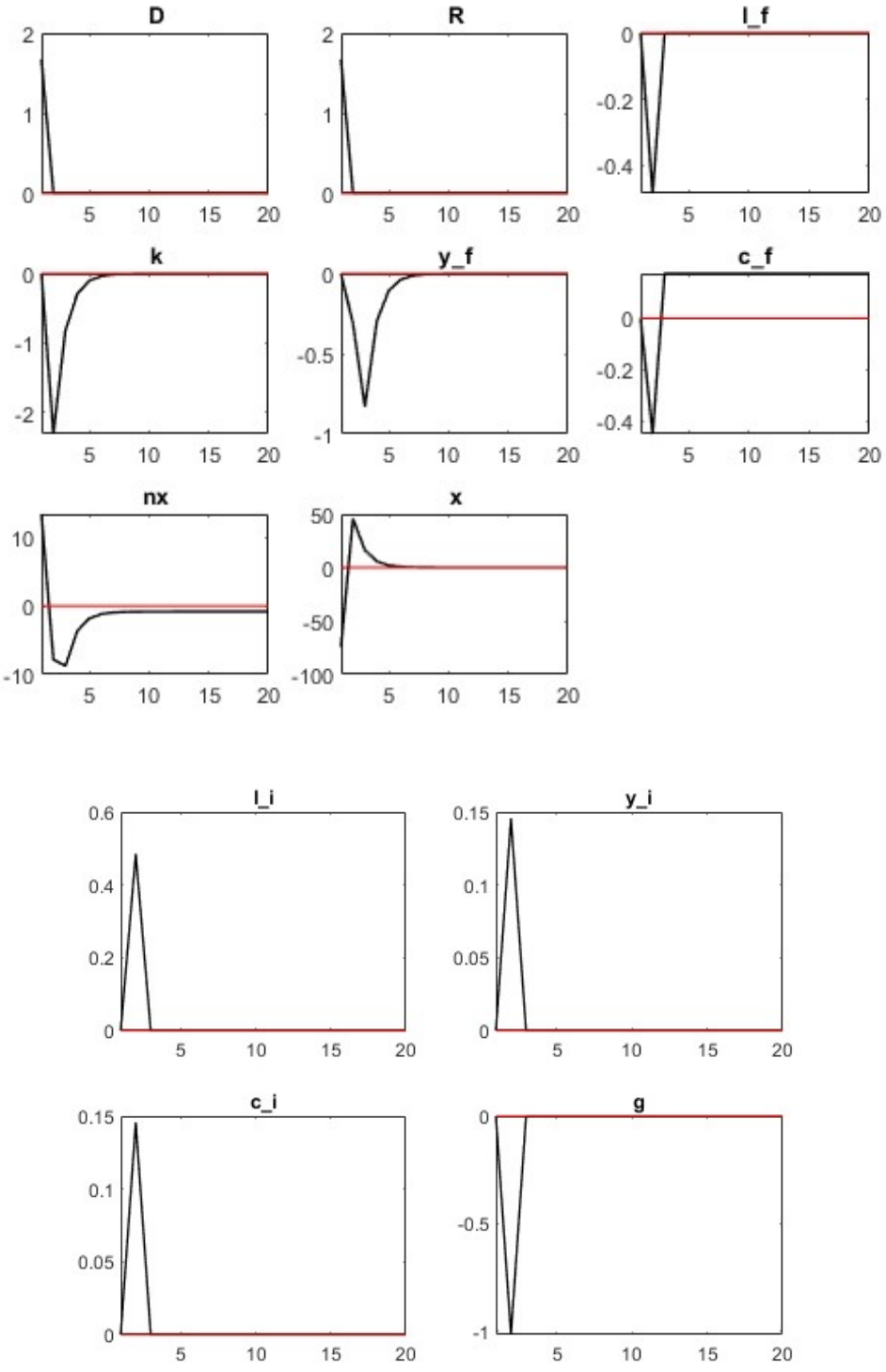


Figure A.20: Effect of Country Risk Shocks: Informal Productivity Induced Case (Working Capital = 1)

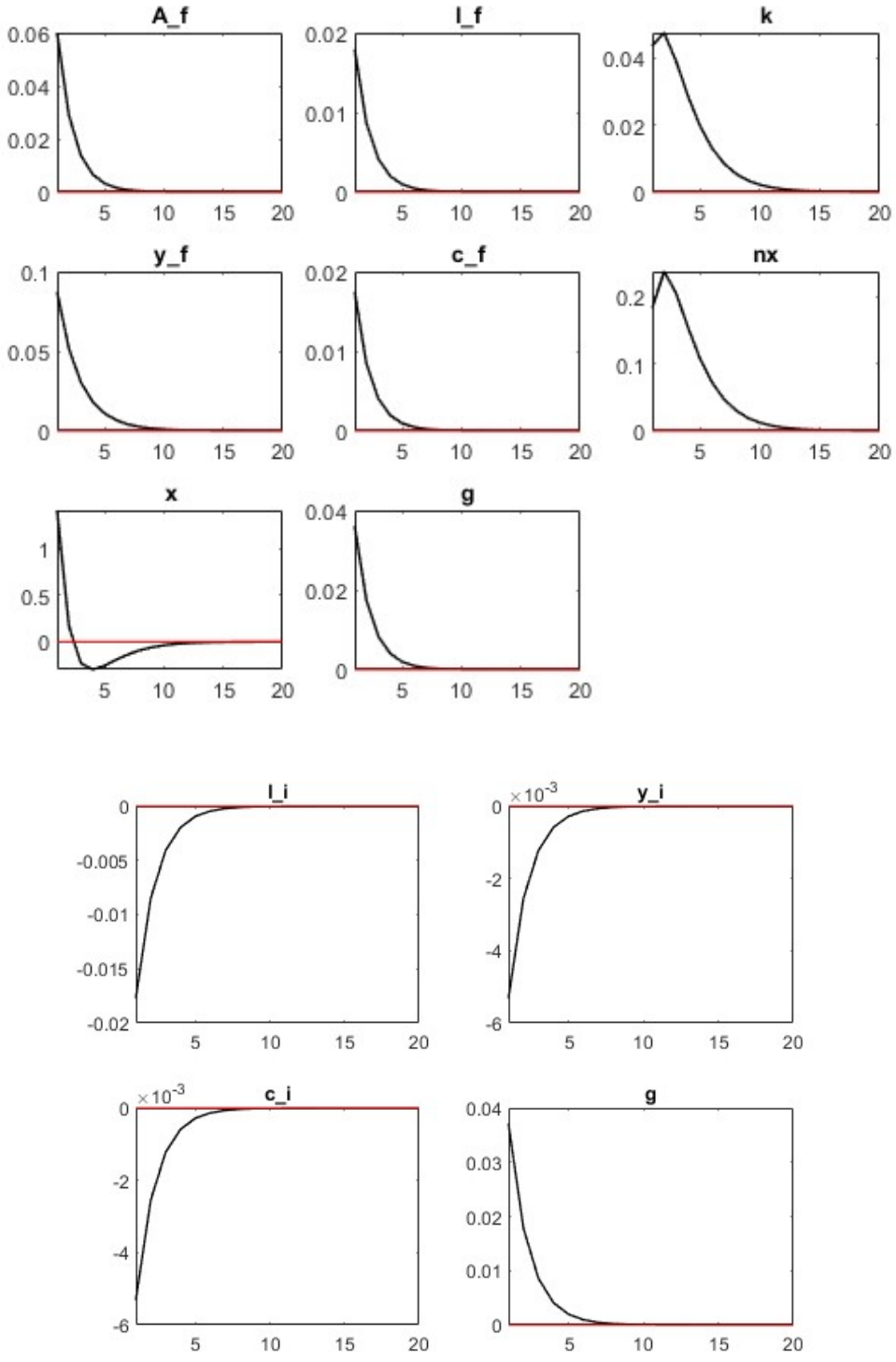


Figure A.21: Effect of Formal Productivity Shocks: Independent Case (Working Capital = 1)

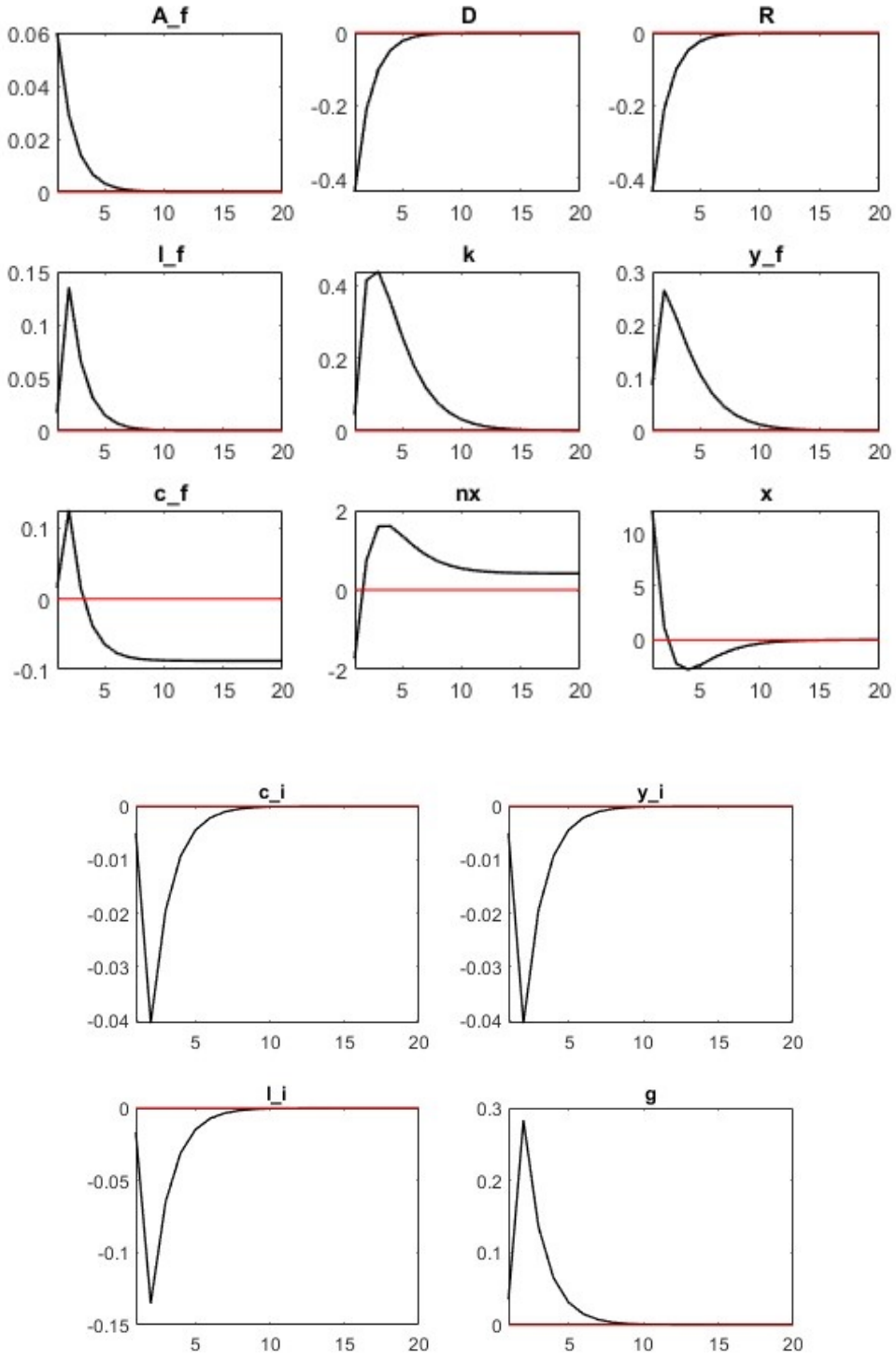


Figure A.22: Effect of Formal Productivity Shocks: Induced Case (Working Capital = 1)

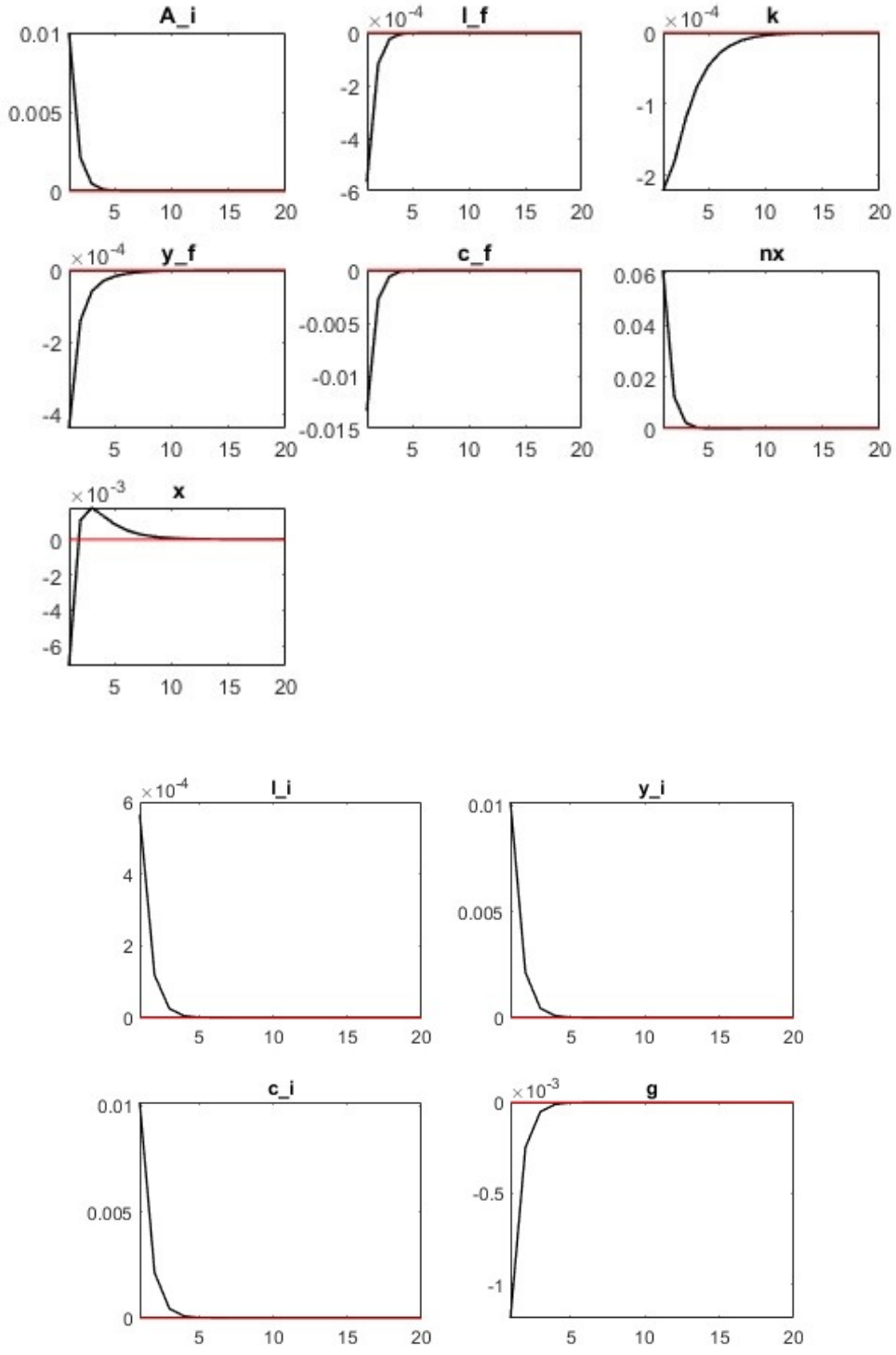


Figure A.23: Effect of Informal Productivity Shocks: Independent Case (Working Capital = 1)

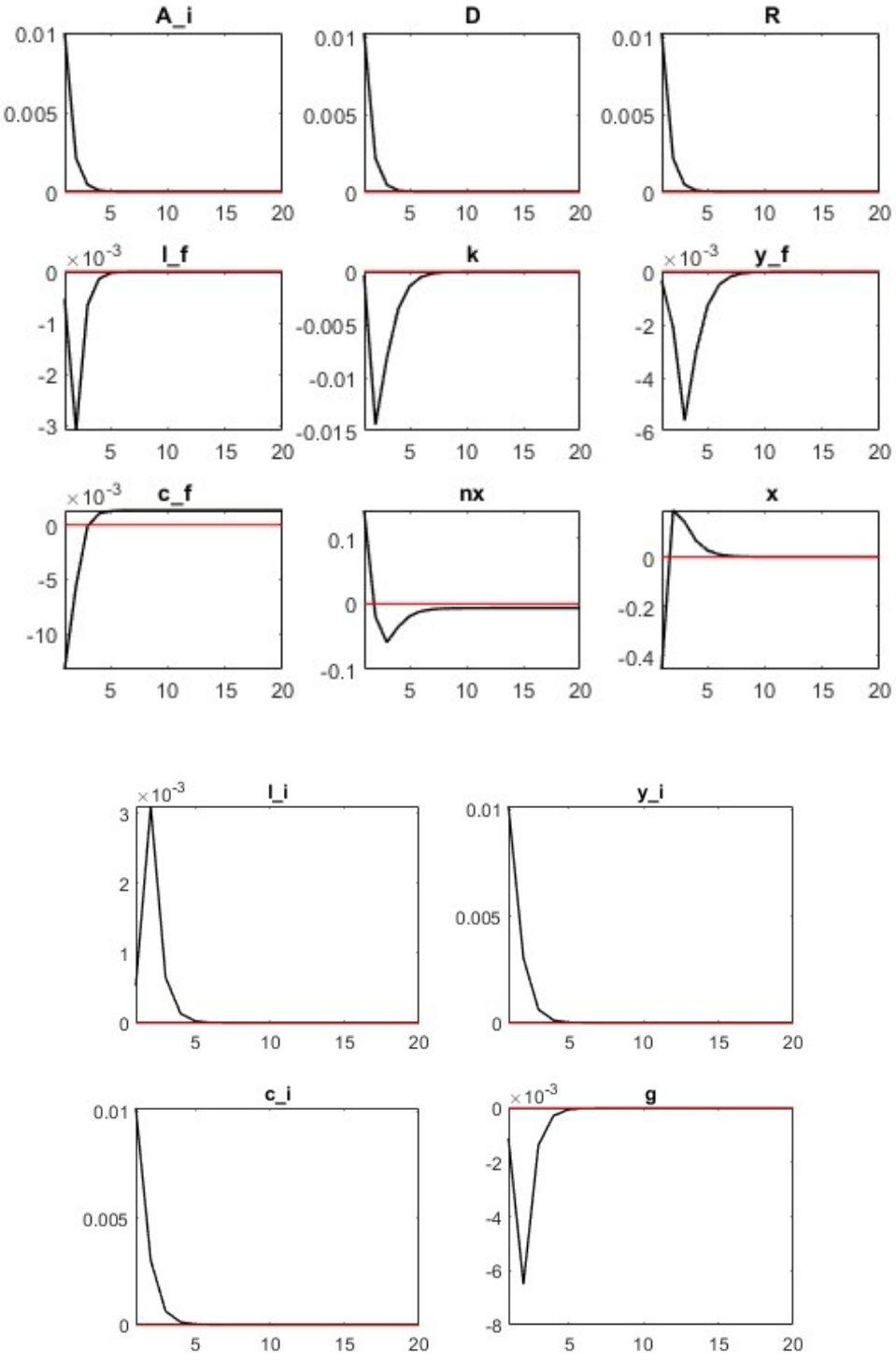


Figure A.24: Effect of Informal Productivity Shocks: Induced Case (Working Capital = 1)

A.5 Robustness Checks

A.5.1 Lag = 2

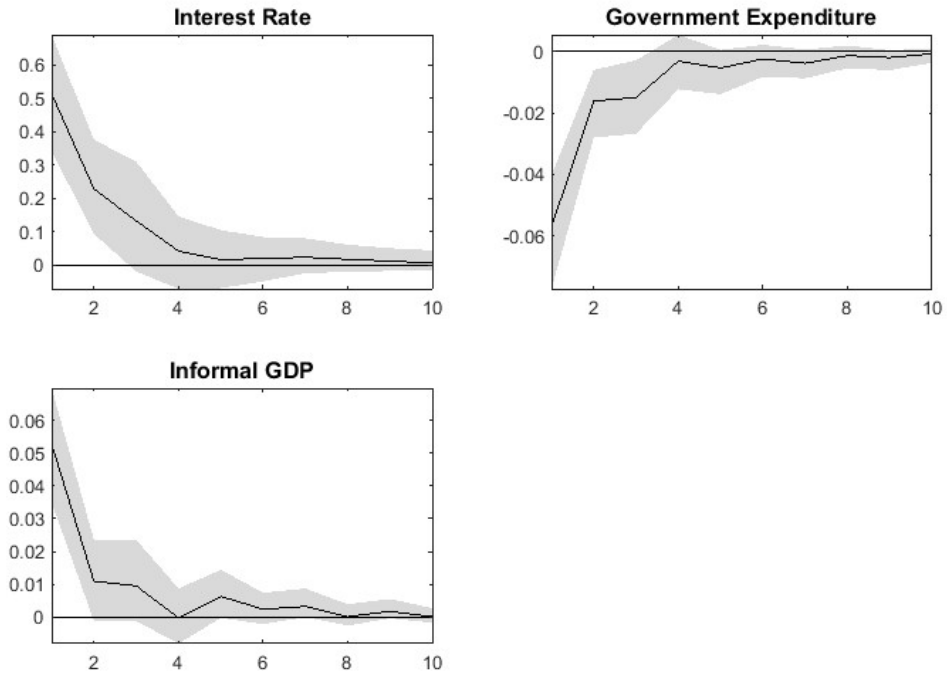


Figure A.25: Effect of Foreign Real Interest Rate Shocks (Empirical Analysis with Lag = 2)

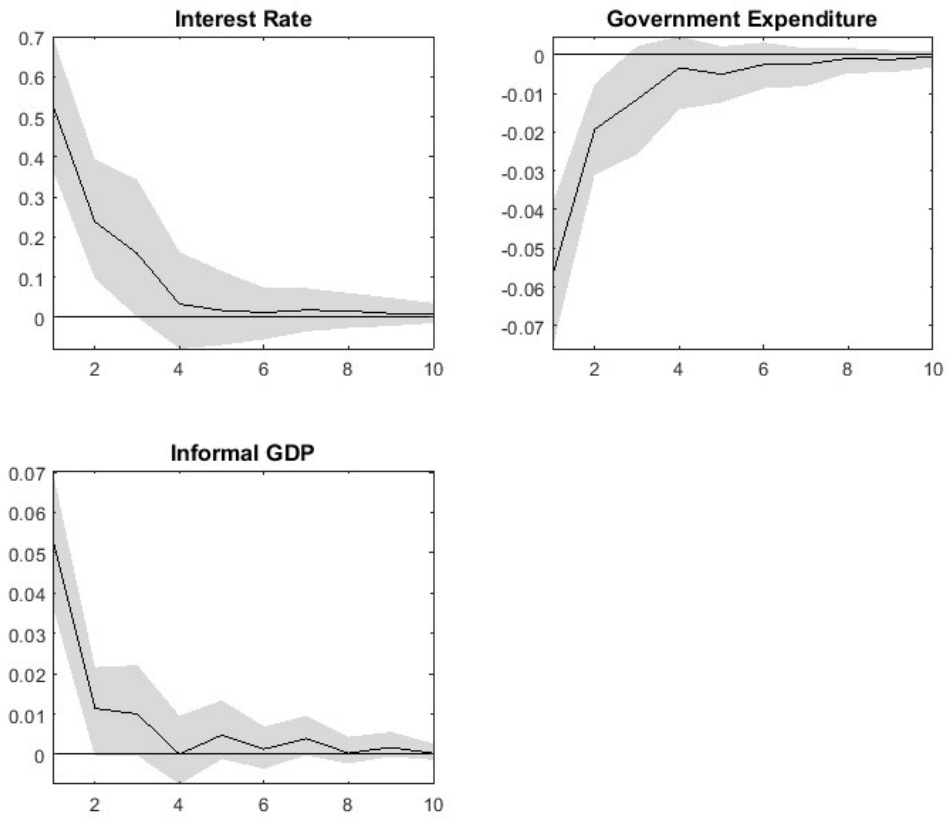


Figure A.26: Effect of Country Risk Shocks (Empirical Analysis with Lag = 2)

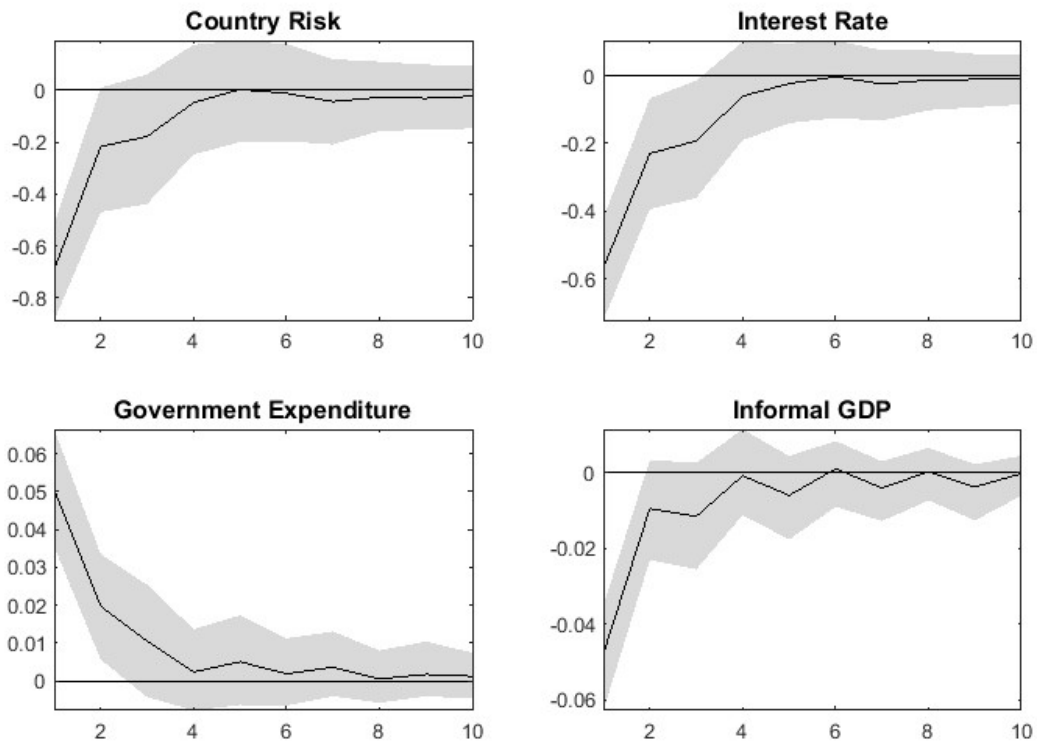


Figure A.27: Effect of Formal Productivity Shocks (Empirical Analysis with Lag = 2)

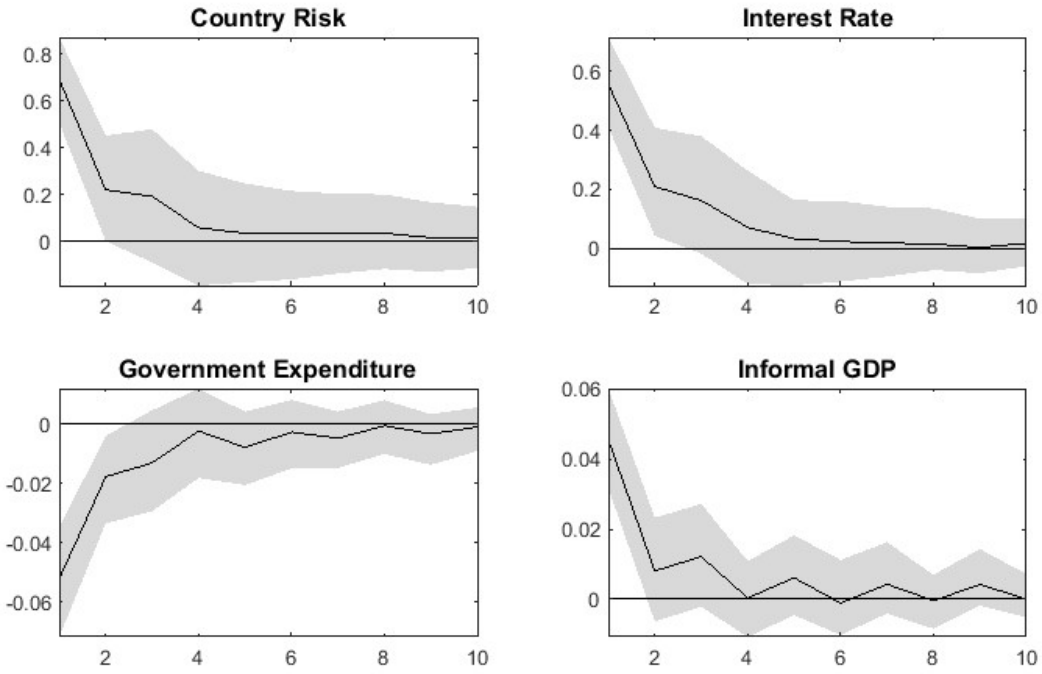


Figure A.28: Effect of Informal Productivity Shocks (Empirical Analysis with Lag = 2)

A.5.2 Lag = 3

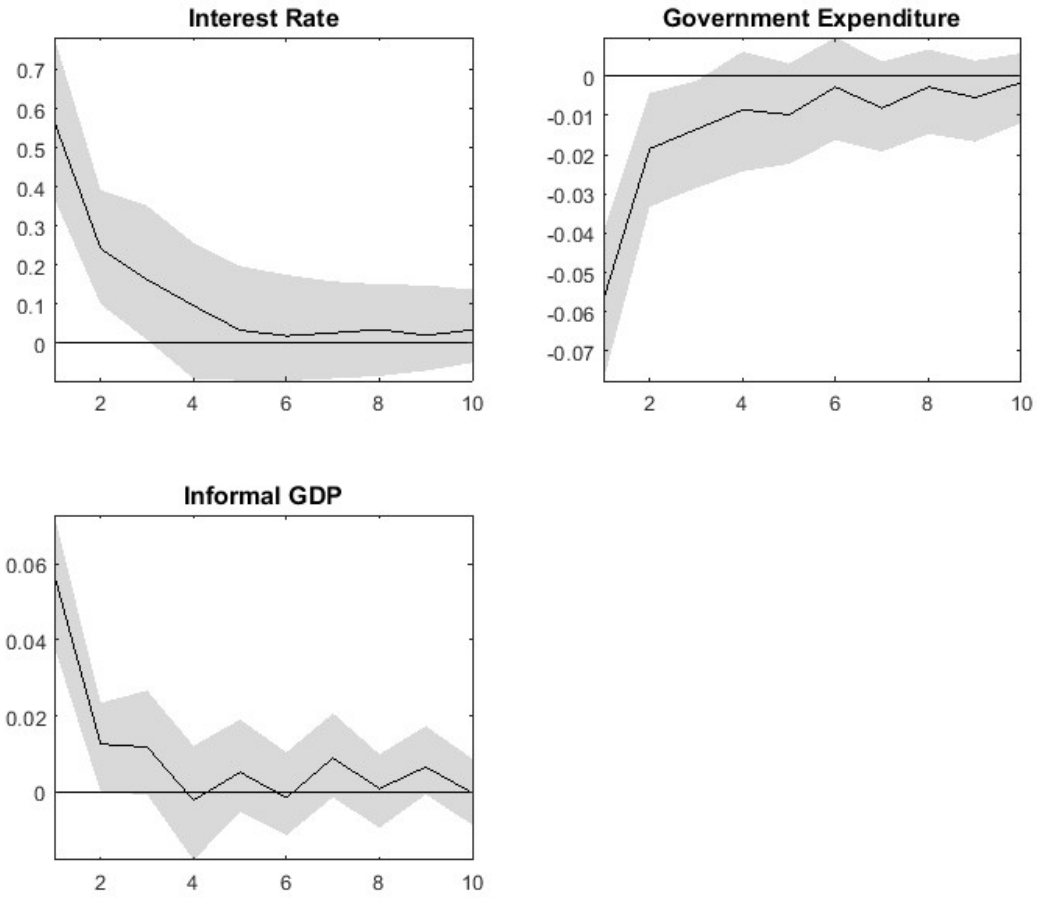


Figure A.29: Effect of Foreign Real Interest Rate Shocks (Empirical Analysis with Lag = 3)

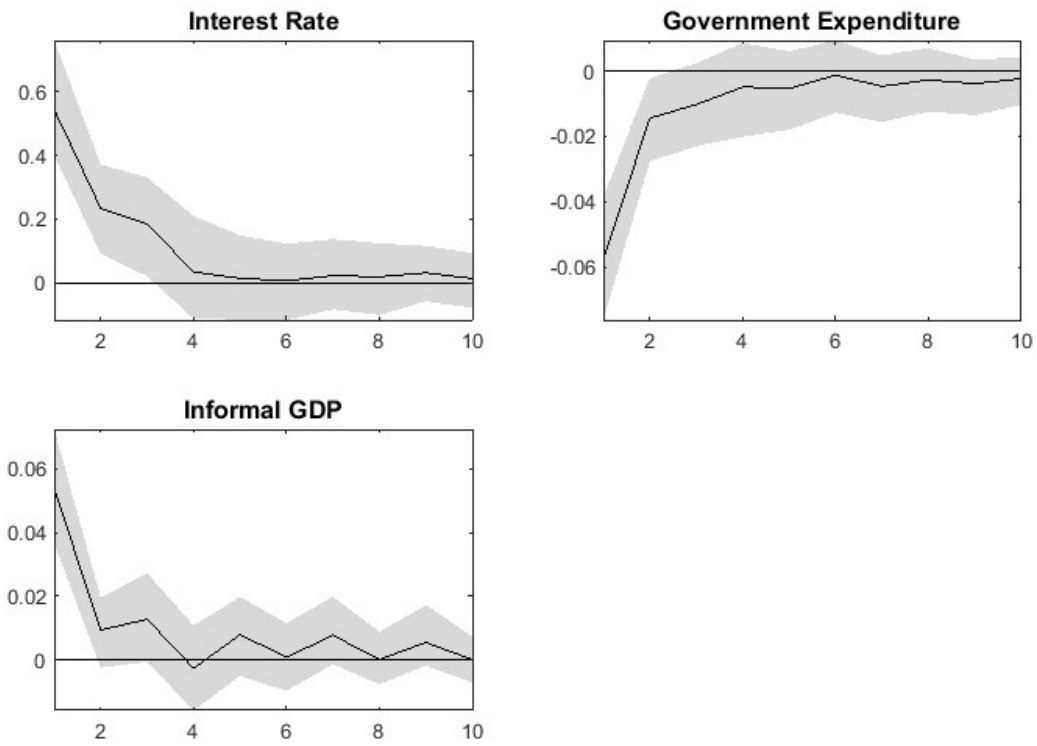


Figure A.30: Effect of Country Risk Shocks (Empirical Analysis with Lag = 3)

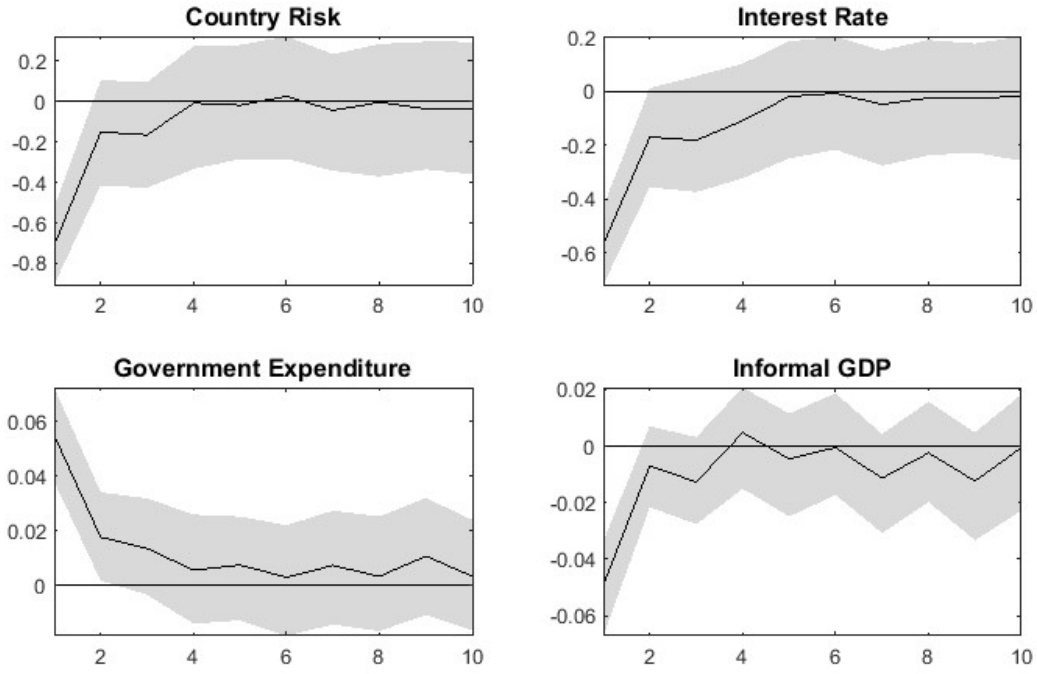


Figure A.31: Effect of Formal Productivity Shocks (Empirical Analysis with Lag = 3)

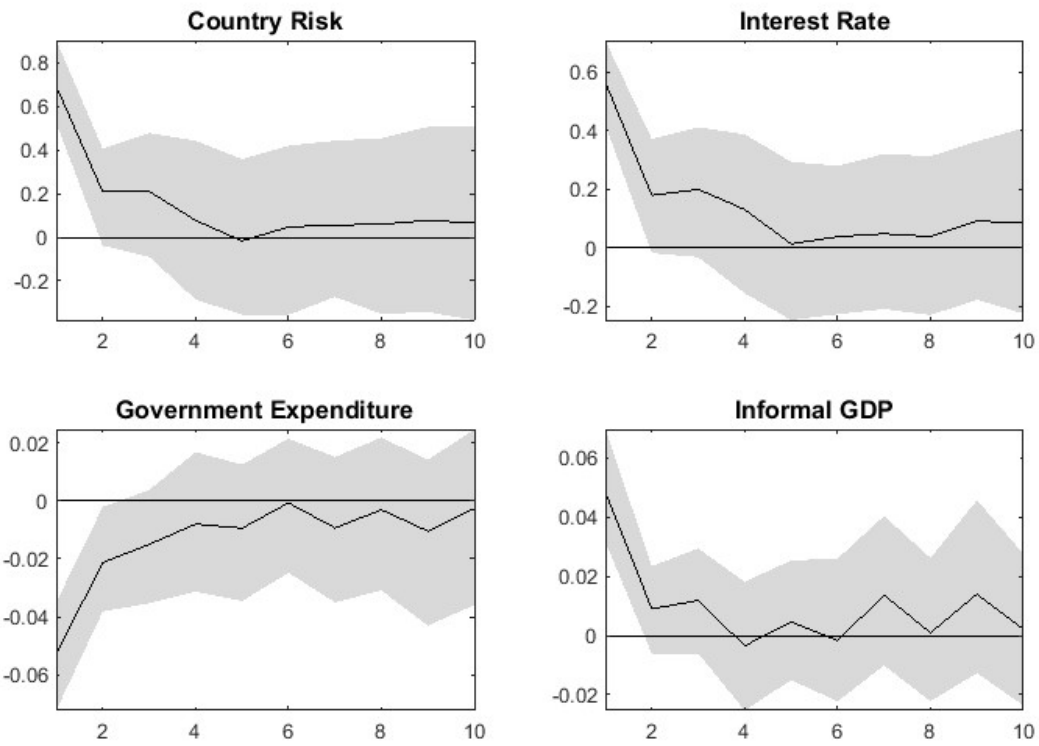


Figure A.32: Effect of Informal Productivity Shocks (Empirical Analysis with Lag = 3)